After Midsem

DATE / / NOTEBOOK

27th sep

In several rigid body dynamic doesn't give you stress [only ...]

* strain changes → determination at micro level
* MEMBERS with non-uniform cross section

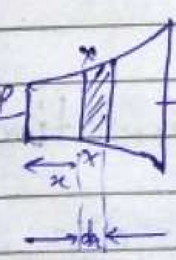
Consider a bar of varying cross section subjected to Tensile ~~pos~~ pass as shown.

→ let a be the cross section area at x.

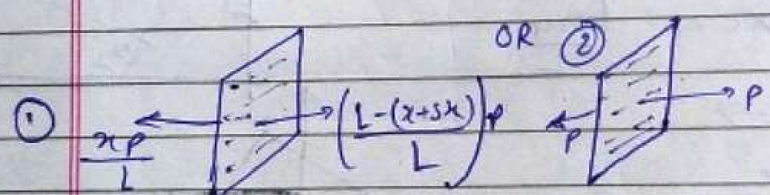
$\sigma = P/A$ $\quad \therefore \frac{P}{A} = E\frac{\delta}{L}$

$\varepsilon = \sigma/E$

Extension of short element δx is

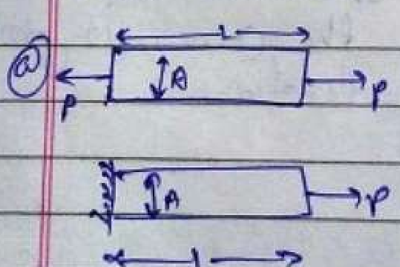


$\Rightarrow \delta = \frac{PL}{AE}$



{But, the first idea was the net force has to be 0 for static equilibrium.}

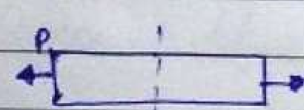
∴ Static equilibrium should exist.



what is the difference in the 2 cases?

Q elongation same or different?

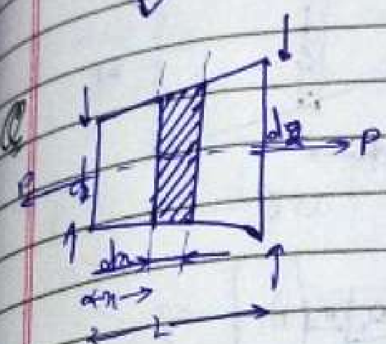
$\therefore \varepsilon = \frac{\delta}{L} = \frac{P\delta}{AL}$



↳ corresponding to support rxn

using boundary condition. In either case ~~str~~ — equilibrium exists.

$d\delta = \frac{P\,dx}{AE}$



area $= \frac{\pi}{4} d_x^2$ $d_x = d_1 + \left(\frac{d_2 - d_1}{L}\right)x$

$\Rightarrow d_x = d_1 + kx$

$\Rightarrow \boxed{x = d_1 x + \frac{kx^2}{2}}$

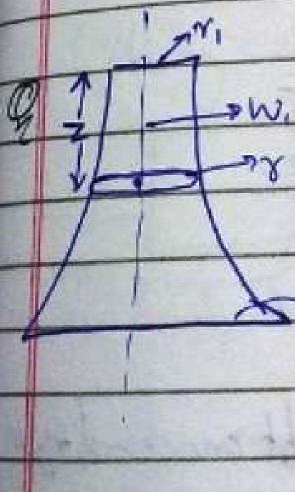
$\Rightarrow \frac{kx^2}{2} + (d_x - 1)x = 0$

$\frac{dx}{x} = \frac{d_1}{x} + \frac{d_2 - d_1}{L}$

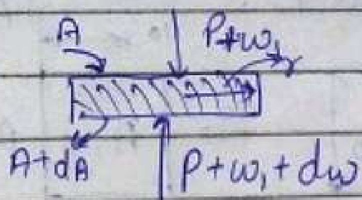
$\log x = d_1 \log x + \left(\frac{d_2 - d_1}{L}\right)x$

$\therefore d\delta = \frac{P\,dx}{\frac{\pi}{4} d_x^2 E}$

$\delta = \frac{4P}{AE} \cdot \frac{kL}{(d_1 + kL)} \Leftarrow \int_0^{\delta} d\delta = \frac{4P}{AE}\int_0^L \frac{dx}{x^2}$



density $\to \rho$



what should be the curvature s.t. stress at every point of cross section is same

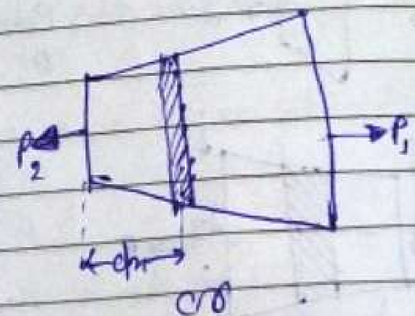
$\Rightarrow \frac{P + w_1}{A} = \frac{P + w_1 + dw}{A + dA}$ $\Rightarrow PA + P\,dA + w_1 A + w_1\,dA = PA + w_1 A + A\,dw$

$\Rightarrow P\,dA + w_1\,dA = w_1 A + A\,dw$ $\Rightarrow (P + w_1)\frac{dA}{A} = dw$

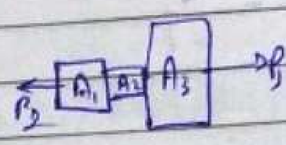
$A = \pi r^2$, $dA = 2\pi r\,dr \Rightarrow \frac{dA}{A} = \frac{2\pi r\,dr}{\pi r^2} \Rightarrow \frac{dA}{A} = \frac{2}{r}dr$

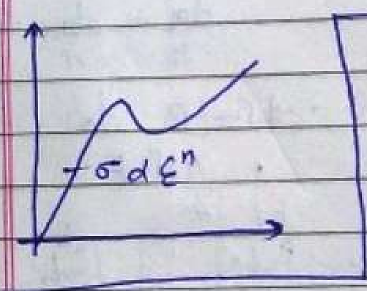
$\Rightarrow dw = \pi r^2 dz = \rho g$

\# $\boxed{\int d\delta = \int \frac{P(x)}{A(x)}\frac{dx}{E}}$



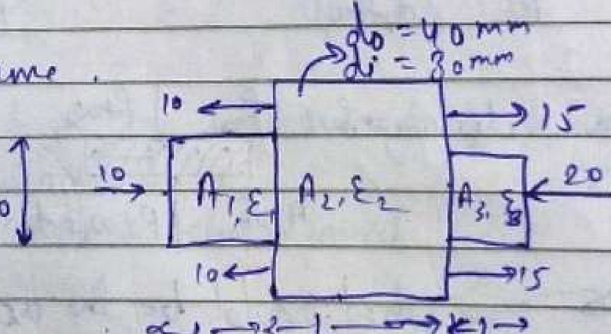
$\delta = \delta_1 + \delta_2 + \delta_3$
$= \frac{P_1L_1}{A_1E_1} + \frac{P_2L_2}{A_2E_2} + \frac{P_3L_3}{A_3E_3}$





Non linear → material
→ geometric

Q E is same.

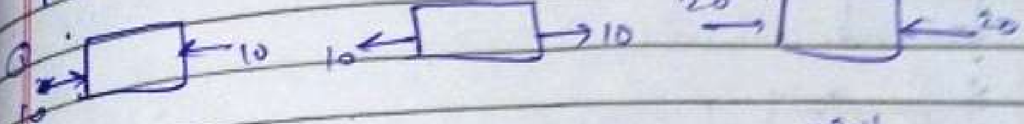


All of mass are cylindrical. The left one and right one solid cylinder and other one is a hollow cylinder.

Step 1: Draw FBD of each part

Step 2: Check whether it is in static equilibrium

F.B.D

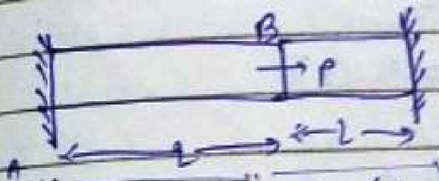


$$\frac{-10L}{\pi(\omega)E_1} \qquad \frac{10L}{\frac{\pi[(10^2-20^2]}{4}E_2} \qquad \frac{-20L}{A(\omega)E_3}$$

$$\frac{P_1L_1}{A_1E_1} = \frac{L}{\pi E}\left[\frac{-12}{40}+\frac{4}{70}\right]$$

Note: Deformation in $\delta = \sum(\delta_1+\delta_2+\delta_3)$

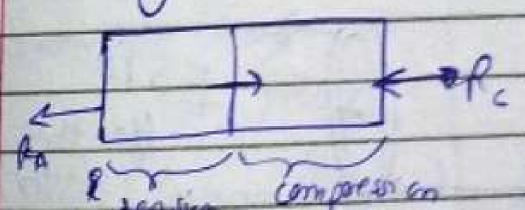
$$= \frac{P_1L_1}{A_1E_1} + \frac{P_2L_2}{A_2E_2} + \frac{P_3L_3}{A_3E_3}$$

Q. 

find a support rxn's at A and at C.

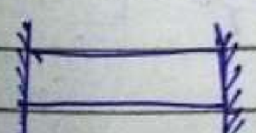
Statically indeterminate problems

Using $\delta_1+\delta_2=0$ Called compatibility Conditions

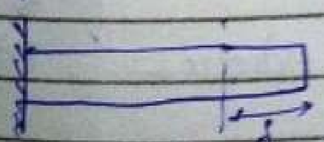


$R_A + R_c = P$

Note: there is always a gap b/w Bridge to avoid resonance other wise Amplitude would go up & cause damage

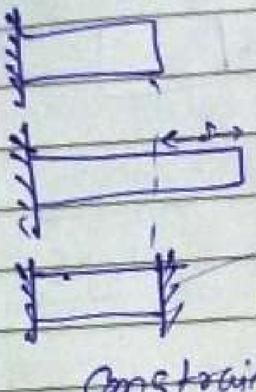
Q. 

$$\left\{\varepsilon_T = \frac{\delta}{L}\right\} = \frac{L\alpha\Delta T}{L}$$



$$\Rightarrow \boxed{\varepsilon_T = \alpha\Delta T}$$

Now if we don't allow it to expand.

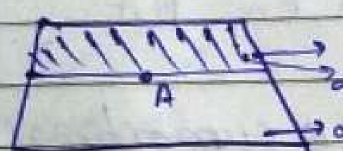


free to expand.

$P = E\alpha\Delta T$ [same as expanding and contracting it by the same Amount.

$\boxed{\sigma_T = E\alpha\Delta T}$

Note
if strain = 0, ⇏ stress = 0 if
strain = 0, strain ≠ 0 why?
if stress confined then no strain



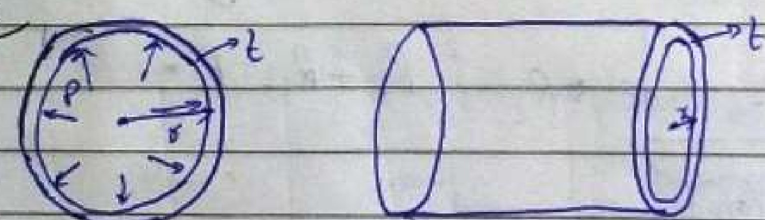
$\varepsilon_1 = \alpha_1 \Delta T$

$\Rightarrow \varepsilon_2 = \alpha_2 \Delta T$

What about point A; what would it follow ε_1 or ε_2.

Delamination would happen or defects (?)

02/3



$\frac{r}{t} > 10$ (for thin cylinder)

⊥ cutting plane

$P_1 = p \times \pi r^2$

$\Rightarrow \sigma \times 2\pi r \times t = p\pi r^2$

$\Rightarrow \boxed{\sigma = \frac{pr}{2t}}$

topics
(1) stress Concentration
(2) Constitutive relation

What will be the principle stresses.

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Constitutive Relation

symmetric.

$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \equiv \sigma_{ij} - \varepsilon_{ij\,kl}\,\varepsilon_{kl} \equiv \sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$

4th order tensor (81 component)

$C_{ijkl} = C_{klij}$

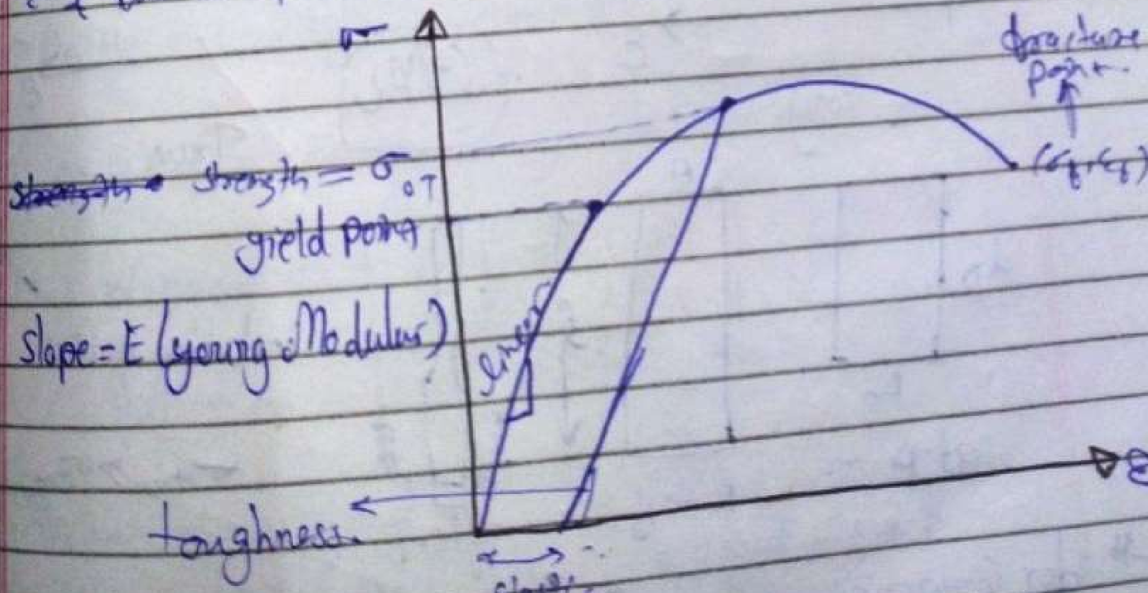
3^4. 81 → isotropic or hydrostatic condition

$$\boxed{W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}} = \frac{1}{2} C_{ijkl}\,\varepsilon_{kl}\varepsilon_{ij} = \frac{1}{2} C_{ijkl}\cdot\varepsilon_{ij}\,\varepsilon_{kl}$$

$$\Rightarrow W = \frac{1}{2}\sigma_{kl}\,\varepsilon_{kl}$$

Q Why we prefer u (displacement) than f (force) while doing stress-strain practical

Ans ∵ u is easy to measure & if we plot the graph b/w σ V/s ε ∵ $\varepsilon = \frac{\Delta l}{l_0}$ So it will be easy to measure the ε & we can plot the graph easily



Elastic Materials →

Nonlinear and linear Elastic Material →

Engineering stress $= \frac{F}{A_0} = \sigma_{eng}$

strain $= \varepsilon_{eng} = \frac{\delta}{l_0} = \frac{l_0 - l_0}{l_0}$

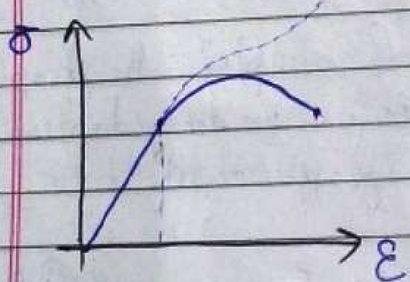
True stress $= \sigma_{true} = \frac{F}{A}$,

— Current values

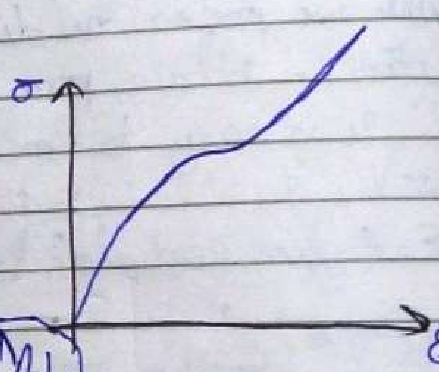
strain $= \varepsilon_{true} = \frac{l - l_0}{l_0}$

$\sigma_T = f(\sigma_{eng})$

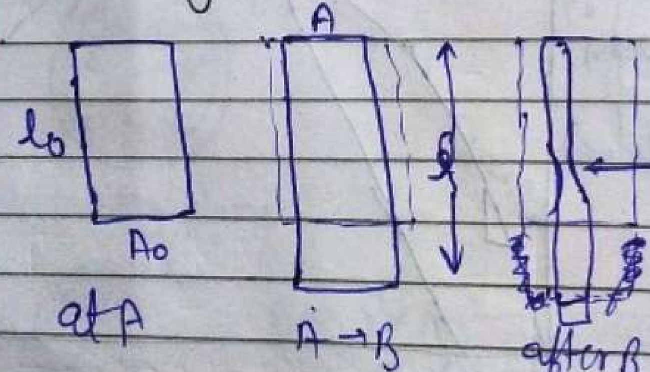
$\varepsilon_T = f(\varepsilon_{eng})$



Eng



True

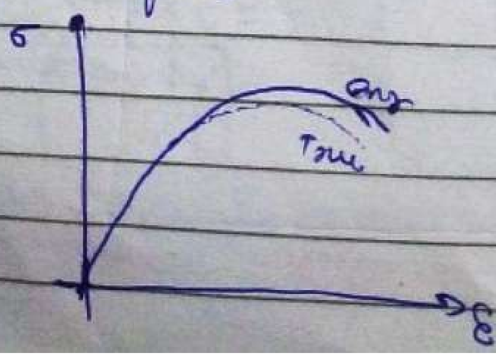


inducing factor

$\sigma_{true} > \sigma_{engineering}$

* for Compression →

$\sigma_{True} < \sigma_{eng}$

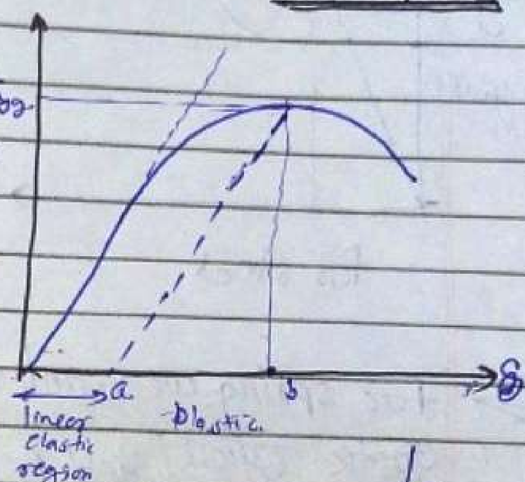


→ shear ~~s~~stress & strain plot
∴ here Area is not changing so True & Engineering graph will remain same.

09/10/2023

0-a → not required strain ε^{PL}

a→b → $\varepsilon^{elastic}$



toughness → What is the amount of energy the material can absorb before failier

(type)

steel has a much higher toughness than a brick.

→ but brick has high strength

→ Brittle material has more strength

Measure of Ductility → ① strength
② toughness

③ % elongation $= \frac{l_f - l_0}{l_0} \times (100\%)$

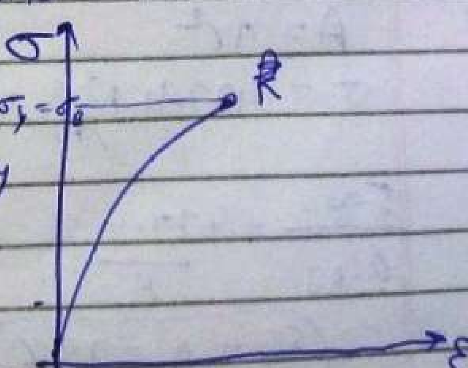
④ % Reduction in Area $= \frac{A_0 - A_f}{A_0} \times 100$

Offset strain = 0.2 %

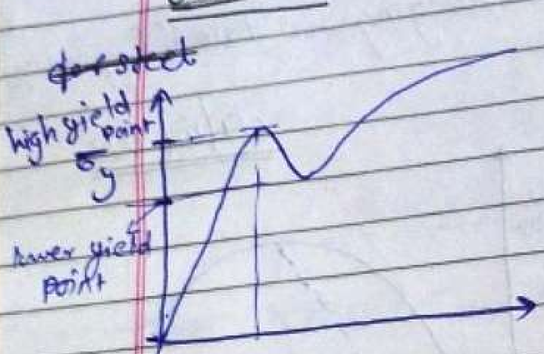
Brittle Material →

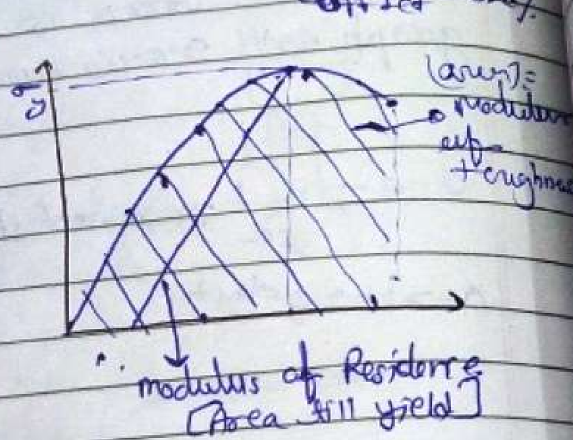
→ little plastic deformation or energy absorption before fracture

→ fractured surface is plane



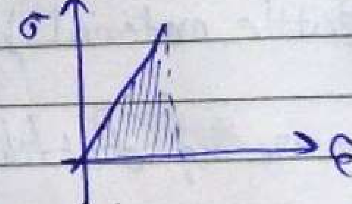
Ductile →





Note For spring we want more energy stored till elastic region [yield point]

Note → When we loaded a Material it increase the yield point or straingth.
if we do it again and again increases brittle nature and decrease ductility.



Strain Energy → $E = \frac{1}{2}(\sigma \varepsilon)$

$F = \sigma \times \Delta A$

Average force $= \frac{0+F}{2} = \frac{F}{2}$

$W = \left(\frac{F}{2}\right) \cdot S$

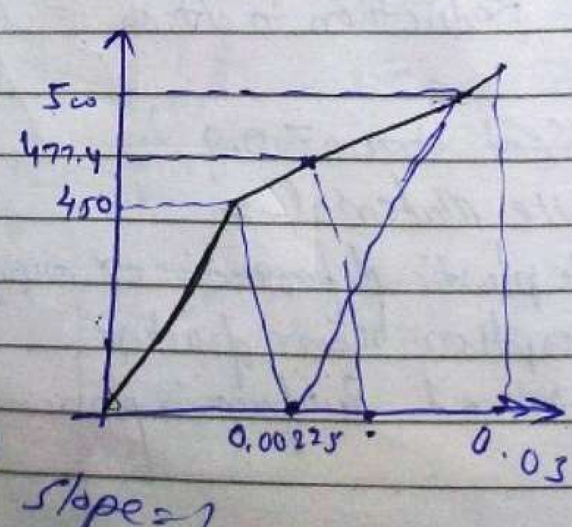
Qu:-
$P = 150\ kN$

$A = \pi r^2$

$\sigma = 477.4\ N/m^2$

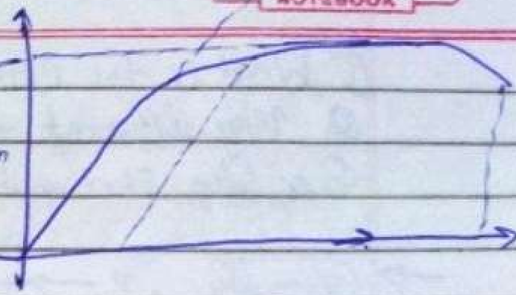
$\frac{500}{0.03} = \frac{477.4}{\varepsilon}$

$\boxed{\varepsilon_T = 0.0286}$



What is happening in elastic region

Material Modulus → [what is happening in plastic region]

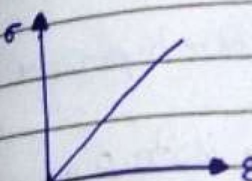


$\boxed{\sigma = E\varepsilon}$ hook's law

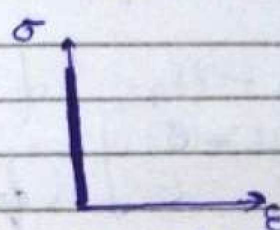
$\sigma \propto f(\varepsilon)$

10/10/2022 Tuesday

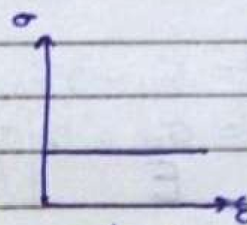
$\sigma \propto f(\varepsilon)$ or $\sigma = f(\varepsilon)$



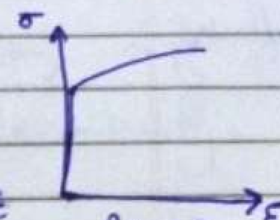
Linear elastic material



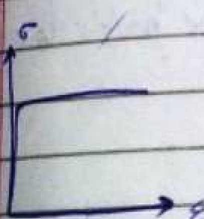
Rigid material



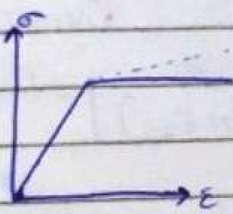
Perfect Plastic (non-strain hardening)



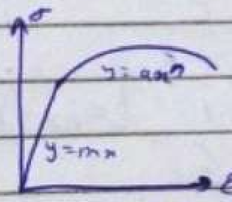
Rigid Plastic (with strain hardening)



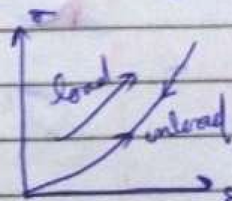
Rigid Plastic [without strain harding]



Elastic Perfectly Plastic Material

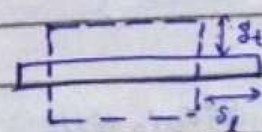


Elastic-Plastic material



Rubber [non-linear elastic material]

Poissions Effect →



$$\boxed{\nu = -\frac{\varepsilon_t}{\varepsilon_l}}$$

for isotropic Material ⇒ $$\boxed{-\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}}$$

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What is the Motivation of poissions effect?

Q What is the Motivation of poissions different poissions ratio?

Q Why different materials have different poissions ratio?

Q or Why steel and Aluminium have different poissions ratio?

→ Generally, $\nu \longrightarrow +ve$

for रबड़ (Bag), $\nu \longrightarrow -ve$

$\sigma_{11} = E\epsilon_{11}$ (not correct) → if there is no poission ratio

effectively → $\epsilon_{11}^{eff} = \epsilon_{11} - \nu\epsilon_{22} - \nu\epsilon_{33}$ [if there is poission ratio]

$= \frac{\sigma_{11}}{E} - \nu\left[\frac{\sigma_{22}}{E} + \frac{\sigma_{33}}{E}\right]$ (for isotropic)

$$\Rightarrow \boxed{\epsilon_{11}^{eff} = \frac{1}{E}\left[\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})\right]}$$ Generalize hook's law

Similarly

$$\epsilon_{22}^{eff} = \frac{1}{E}\left[\sigma_{22} - \nu(\sigma_{11} + \sigma_{33})\right]$$

$$\epsilon_{33}^{eff} = \frac{1}{E}\left[\sigma_{33} - \nu(\sigma_{11} + \sigma_{22})\right]$$

H.C.

$\sigma_{11} = f(\epsilon_{11}^{eff}, \epsilon_{22}^{eff}, \epsilon_{33}^{eff})$

let $\epsilon_{ii}^{eff} \longrightarrow \epsilon_{ii}$

* for plain strain ⇒ if $\epsilon_{33} = 0 \Rightarrow \frac{1}{E}\left[\sigma_{33} - \nu(\sigma_{22} + \sigma_{11})\right] = 0$

$$\Rightarrow \boxed{\sigma_{33} = \nu(\sigma_{11} + \sigma_{22})}$$

* for plain stress ⇒ $\sigma_{33} = 0 \Rightarrow \epsilon_{33} \neq 0$

Cont

In Matrix form → (stiffness ~~of~~ Matrix)

$$\boxed{\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{11} + \nu(\varepsilon_{22}+\varepsilon_{33})\right]}$$

$\because E = 2G(1+\nu)$ ~~and~~ → $(1+\nu) > 0 \Rightarrow \nu > -1$

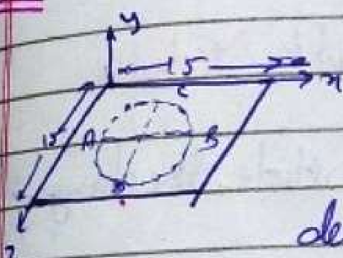
$E = 3k(1-2\nu)$ → $1-2\nu > 0 \Rightarrow \nu < 1/2$

Range of ν = ?

$\boxed{-1 < \nu < 1/2}$ theoritically

$\boxed{0 < \nu < 0.49}$. practically

Q:-



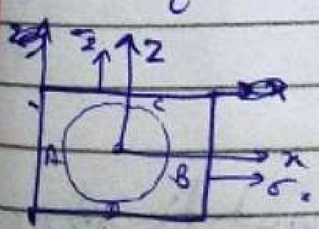
$d = -a$, $t = 3/4$, $\sigma_x = 12$, $\sigma_z = 20$

$E = 10 \times 10^6$, $\nu = 1/3$

determine change in ① length and Diameter AB
② length the Dia CD
③ $t' = ?$ ④ Volume = ?

Solution → $\because \sigma_y = 0 \Rightarrow$ plane stress

$$\Rightarrow \varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\sigma_{zz}\right] = 10^{-7}\left[12 - \frac{1}{3}\times 20\right] = 5.33\times 10^{-7}$$



$$\frac{l_{AB} - (l_0)_{AB}}{(l_0)_{AB}} = \varepsilon_{xx} \Rightarrow l_{AB} =$$

Ans

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② $\varepsilon_{zz} = \frac{1}{E}[\sigma_{zz} - \nu\sigma_{xx}] = \frac{1}{E}\left(20 - \frac{1}{3}\times 12\right) = 13.33\times10^{-}$

$\Rightarrow \frac{l_{CD} - (l_0)_{CD}}{(l_0)_{CD}} = \varepsilon_{zz} \Rightarrow l_{CD} =$

③ $\varepsilon_{yy} = \frac{1}{E}[0 - \nu(\sigma_{xx} + \sigma_{zz})] = -\frac{1}{E}\left[\frac{1}{3}\times 32\right] = -10.67\times10^{-7}$

$\Rightarrow \frac{t' - t}{t} = \varepsilon_{yy} \Rightarrow t' = 0.7499 \; 0.75 = 3/4$

④

11/10/2023

Dilatation: Bulk Modulus.

→ Relative to the unstress state, the change in Volume is

$V_1 = 1\times1\times1 = 1,\; V_2 = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$

$\Rightarrow V_2 = 1 + \varepsilon_x + \varepsilon_y + \varepsilon_z$ [$\varepsilon_x, \varepsilon_y$ and ε_z are very sm

$\Rightarrow \Delta V = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] + \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] + \frac{1}{E}[\sigma_z - \nu\sigma$

$= \frac{1}{E}[(1-2\nu)(\sigma_x + \sigma_y + \sigma_z)]$

$\Delta V = \frac{1}{E}[(1-2\nu)(3p)]$ → hydrostatic stress.

where $p = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$

$$\Rightarrow \frac{P}{\Delta V} = \frac{E}{3(1-2\nu)}$$

$k \rightarrow$ Bulk modulus

$$\Rightarrow k = \frac{E}{3(1-2\nu)}$$

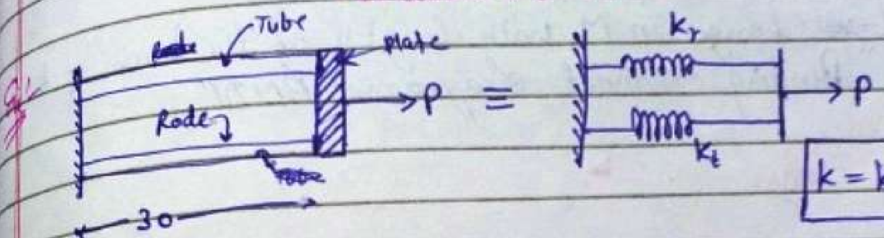
$$\Rightarrow \boxed{E = 3k(1-2\nu)} \; ; k > 0$$

$$\boxed{0 < \nu < 1/2}$$

Bulk modulus $\rightarrow$ resistance to compression

Residual stresses

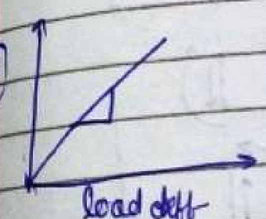
A_r ✓ A_t ✓
ε_r ✓ ε_t ✓
γ_r ✓ γ_t ✓



$$\boxed{k = k_r + k_t}$$

Stiffness $\boxed{k = \frac{EA}{L}}$

(a)



$A_{total} = 0.175\, m^2$, $P = 5.7\ kPa$

$\boxed{P_r + P_t = 5.7}$ — (1)

$\sigma = 32.5 \frac{kPa}{in^2}$

$$\frac{EA}{l} = \frac{E_r A_r}{l_r} + \frac{E_t A_t}{l_t}$$

$\boxed{P_r + P_t = 5.7}$

$\delta_r = \delta_t$

$$\Rightarrow \frac{P_r}{k_r} = \frac{P_t}{k_t}$$

$$\Rightarrow \frac{P_r L_r}{E_r A_r} = \frac{P_t L_t}{E_t A_t}$$

$$\Rightarrow \boxed{\frac{P_r}{P_t} = 1.5} \text{ — (2)}$$

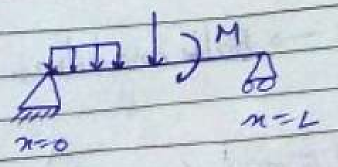
from (1) and (2)

$\boxed{P_r = 3.2 , P_t = 2.28}$

$\Rightarrow \delta_r = \frac{P_r L_r}{E_r A_r} = \ldots$ & $\delta_t = \frac{P_t L_t}{E_t A_t} \ldots$

Elongation $= \min(\delta_r, \delta_t)$

Chapter (3) Bending. [Elastic deformation in Bending]



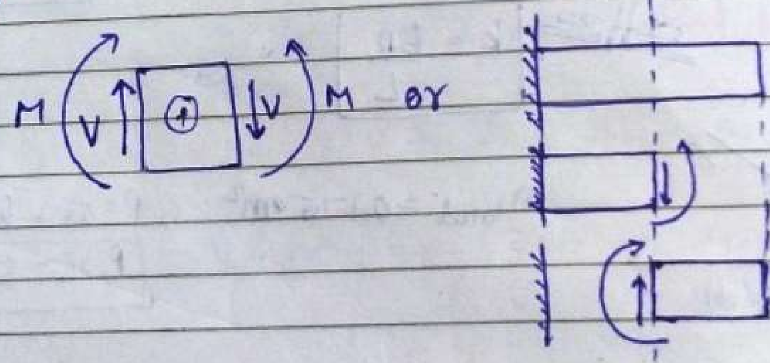
shear force → V
Bending Moment → M

step 1 → draw free body Diagrame
step 2 → where is V & M are max.

Note → graph b/w shear force and length of the beam is called "shear force diagrame (SFD)".

Note → ~~Grau~~ change in M with length of the beam is called as "Bending Moment diagrame (BMD)".

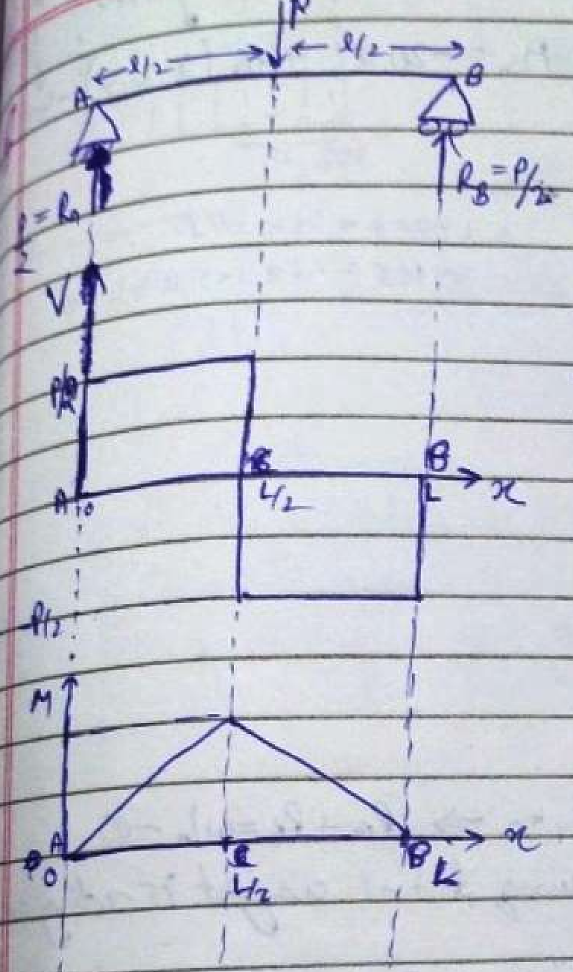
sign Convention →



for +ve ~~Load~~

Support →

Pin joint	Δ	R_1, ~~R_2~~ and R_2
Roller		R_2
Fix		R_1, R_2 and M



for SFD

$\Rightarrow R_A + R_B = P$ & $\Rightarrow \sum M_B = 0$

$\Rightarrow R_A \times L - P \times \frac{L}{2} = 0$

$\Rightarrow \boxed{R_A = R_B = P/2}$

→ at point C I can say the jump happed due to concentric load.

For BMD,

let M_x is Moment at any crosssection x

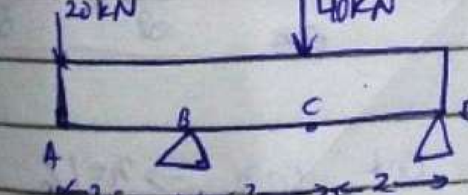
$\boxed{M_x = \underbrace{R_A \times x}_{\text{for AC}} - P(x - L/2)}$

there is a term until cross the next term.

$0 < x < L/2 \Rightarrow M = \frac{P}{2} x$

at $x = \frac{3}{4}L$, $M = \frac{P}{2} \times \frac{3L}{4} - P\left(\frac{L}{4}\right) \Rightarrow M = PL/8$

at $x = L \Rightarrow M = \frac{P}{2}L - \frac{P}{2}L \Rightarrow M = 0$ [draw MBD]



$\sum F_y = 0 \Rightarrow -20 + R_B - 40 + R_D = 0$

$\Rightarrow \boxed{R_B + R_D = 60\text{ kN}}$

$\sum M_D = 0$

$\Rightarrow -20 \times 7.5 + R_B \times 5 - 40 \times 2 = 0$

$\Rightarrow \boxed{R_B = 46}$

$\Rightarrow \boxed{R_D = 14}$

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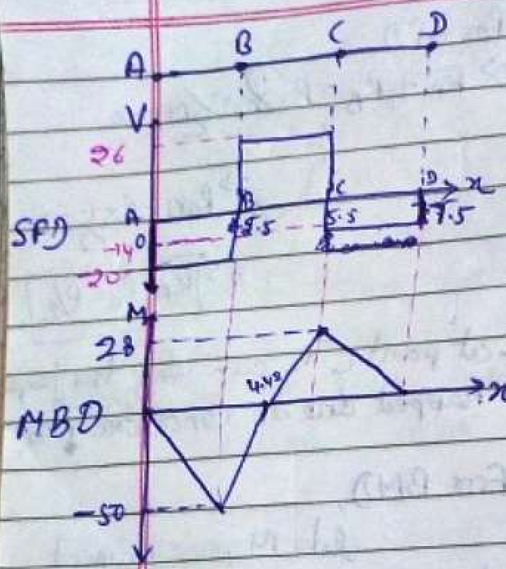
Acting when $x>2.5$ Acting when $x>5.5$

$M_x = -20x \; ; \; +46(x-2.5) \; ; \; -40(x-5.5)$



$\frac{dM_x}{dx} = 0$

$0 = -20x + 46x - 115$

$\Rightarrow 115 = 26x \Rightarrow x = 4.42$

Distributed load →

$\sum F_y = 0 \Rightarrow R_A + R_B = wL$ — ①

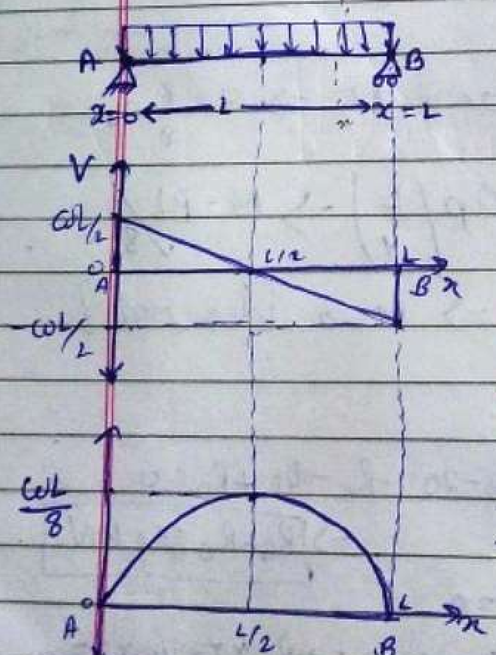
assuming total weight is acting on COM

$\Rightarrow \sum M_B = R_A \times L - (wL)L/2 \Rightarrow \boxed{R_A = \frac{wL}{2} = R_B}$

for SFD $\Rightarrow V_x = \frac{wL}{2} - wx$

for MBD $\Rightarrow M_x = \frac{wL}{2}x - \frac{wx^2}{2}$

?



max

$\frac{dM}{dx} = \frac{wL}{2} - wx \Rightarrow x = L/2$

$(M_x)_{max}\Big|_{x=L/2} = \frac{wL^2}{8}$

?

$w = \frac{1}{2} \times 4 \times 400 = 800$

$Q = 1000 + 3w \times 4 = 2200$

$\frac{1000}{2200} =$

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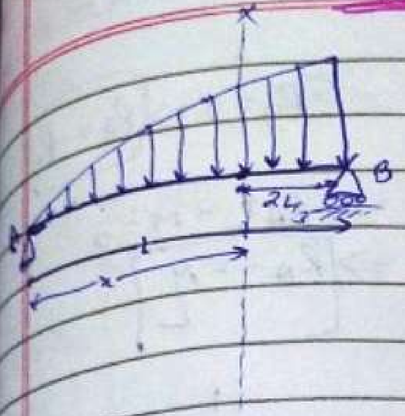
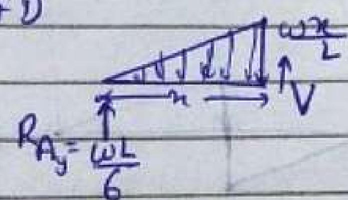


$\Sigma F_x = 0 \Rightarrow R_A = 0,\ \Sigma F_y = 0 \Rightarrow \boxed{R_A + R_B = \frac{\omega L}{2}}$

$\Sigma M_A = L \times R_B + \frac{2L}{3}\frac{\omega L}{2}$

$\Rightarrow L \times R_B = \frac{\omega L^2}{3} \Rightarrow \boxed{R_B = \frac{\omega L}{6}}\ \boxed{R_A = \frac{\omega L}{6}}$

for SFD



$\Rightarrow$ S.F. $\Rightarrow \frac{\omega l}{6}\uparrow + \frac{1}{2} * \frac{\omega x^2}{l}\downarrow + V\uparrow = 0$

$\Rightarrow \boxed{V = \frac{-\omega l}{6} + \frac{\omega x^2}{2l}}$

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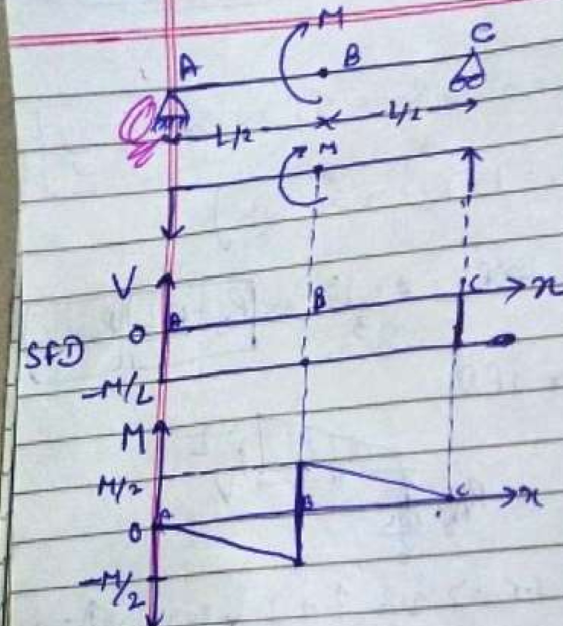


$\Sigma F_x = 0$, $\Sigma F_y = 0 \Rightarrow \boxed{R_A = -R_B}$

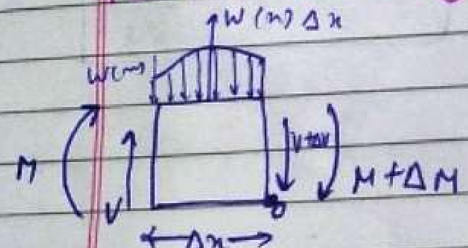
$M_C = 0 \Rightarrow -R_A L + M = 0$

$\Rightarrow \boxed{R_A = -\frac{M}{L}}$

$M_x = -\frac{M}{L}x + M$

always Value of the jump is equal to applied Moment.

Relation ~~Between~~ among load, shear and Bending Moment



$\Sigma F_x = 0$, $\Delta \Sigma F_y = 0$

$\Rightarrow V - (V+\Delta V) - w(x)\Delta x = 0$

$\Rightarrow \frac{\Delta V}{\Delta x} = -w(x)$

if $x \to 0$

$\boxed{\frac{dV}{dx} = -w(x)}$ → density of distributed load.

$\underline{dV = -w(x)dx}$ $\Rightarrow \Delta V = F$ [Concentrated load]

Σ applied force

$\Sigma M_o = 0 \Rightarrow V\Delta x + M - w(x)\Delta x\left(\frac{\Delta x}{2}\right) - (M+\Delta M) = 0$

and $\Delta x \to 0$

$\Rightarrow \Delta M = V\Delta x$

$\Rightarrow \boxed{\frac{dM}{dx} = V(x)}$

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OR, $V = -\int W\,dx$ and $M = \int V\,dx$

$R_A + R_c = 15 + 5 \times 5$

$\Rightarrow \boxed{R_A + R_c = 40}$

$M_C \Rightarrow 80 + R_A \times 10 - 15 \times 5 - 25 \times 2.5 = 0$

$\boxed{\cancel{R_B = 40 - \omega}}$, $\boxed{R_c = 34.25}$

$\boxed{R_A = 5.75}$

?

$M_x = (80 + R_A x) \,|\, -15(x-5) - \frac{5(x-5)^2}{2}$

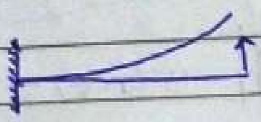
$V_x = R_A \,|\, -15 - 5(x-5)$

$= 5.75 \,|\, -15 - 5(x-5)$

← Bending stresses →

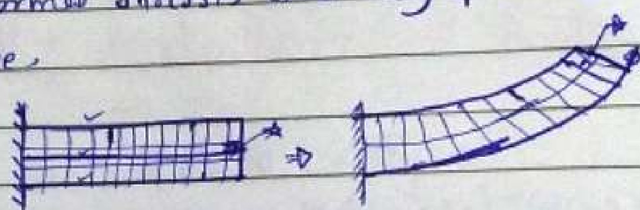
→ The load acting on a beam cause the beam to bend-deforming its axis into a curve.

→ we are interested in center line only.



Bending stresses are normal stresses and vary from one surface to another surface.

What happens here...



the center line does not deform so the section where the force is ⊥ does not deform and also plane [where the force acting] rotation is not allowed during shear or torsion forces.

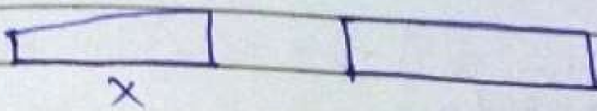
or we can say →
① Compressive ② tensile ③ Plane section remain plane
④ center line (N A) → No deformation ⑤ No rotation at cross-section

Bending of straight member → [Note → we want symmetry. → very Imp. to draw the axis]

→ Beams having a cross-sectional area that is symmetric w.r. to an axis.

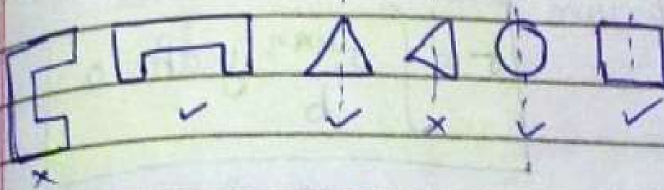
→ Bending moment is applied about an axis ⊥ to this axis of symmetry

→ Prizmatic beam → Cross-section area is not change.



* cross-section area should same → about x-axis.
  symmetry line → y-axis
  applying load → about z-axis. Moment $= M_z = 0\hat{i} + 0\hat{j} + M_z\hat{k}$

→ If we changes the axis the eqn will get change.

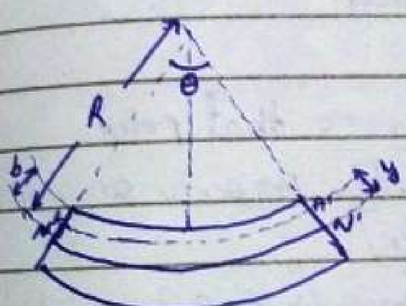


Step 1 Calculating strain first →

$NN' = R\theta$ if $\theta \to$ small.

$AA' = (R-y)\theta$

$\delta = \underbrace{[R-y]\theta}_{\text{deformed length of AA'}} - \underbrace{R\theta}_{\text{not deformed length of AA'}}$



$\varepsilon = \dfrac{\delta}{R\theta} = \dfrac{(R-y)\theta - R\theta}{R\theta} = \dfrac{-y}{R} = \dfrac{l - l_0}{l_0}$

$\left\{ \text{if } \begin{array}{l} R\theta = l_0 \\ (R-y)\theta = l \end{array} \right\}$

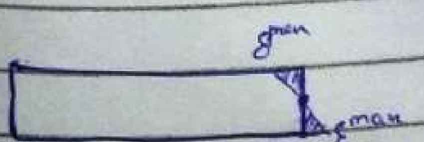
★ $\boxed{\varepsilon^{max} = \dfrac{-b}{R}}$ if $y = b$

⇒ $\boxed{\varepsilon = +\dfrac{y}{b}\,\varepsilon^{max}}$

where y is the distance from the Natural axis.

y → +ve towards tensile area

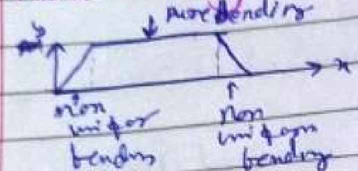
y → -ve towards compressive areas



Step 2 $\sigma = \varepsilon E$

(under a constant moment)

Pure bending and non-uniform bending



stresses due to bending

$\sigma = \frac{\pm y}{b} E \varepsilon_{max} \Rightarrow \boxed{\sigma = \frac{y}{b} \sigma_{max}}$

Q How to determine position of natural axis??

Sol: for static equilibrium $\boxed{\sigma = \int \frac{\sigma^{max}}{b} y \, dA = 0}$

NA

$\int y\,dA = 0$ [first moment of the area about the NA]

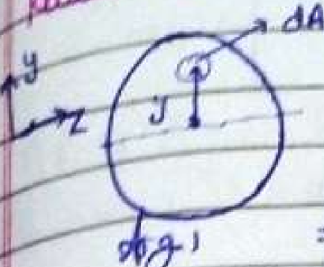
$\int y^2 \, dA \neq 0$ [second moment]

→ there is no unbalance point w.r to that point
[here we are talking about Geometric property or w.r.to area not w.r.t. to mass [gravity center]

→ the $\int y\,dA$ will be on the symmetric lines OR area balance

in term of submission $\boxed{\bar{y} = \sum_{i=1}^{m} y_i A_i / \sum A_i}$

Relation b/w stress and Movement [or Moment balance] →



$$dM = \int \sigma \, dA \times y = \frac{\sigma_m}{b} \underbrace{\int y^2 dA}_{\text{Second Moment}} \; (= I)$$

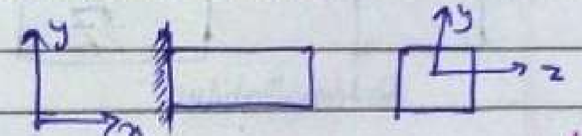
$$\Rightarrow dM = \frac{\sigma^m}{b} I \Rightarrow \boxed{\sigma^m = \frac{bM}{I}}$$

$$\Rightarrow \boxed{\sigma = \frac{\sigma^m y}{b}} \qquad \Rightarrow \boxed{\sigma = \frac{M}{I} y} \rightarrow \text{flexure formula}$$

Q What about axis?

→ In fig. 1, M is taken about z axis

$$\Rightarrow \boxed{\sigma_{xx} = \frac{M_z y}{I_z}}$$



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← flexure formula →

$$\sigma_x = \frac{-My}{I}$$

$$\Rightarrow df = \sigma dA \qquad \Rightarrow \int dM = \int \sigma \, dA \times y$$

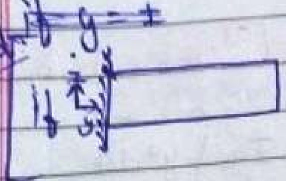
$$= \frac{y \sigma_{max} dA y}{b}$$

$$= \frac{\sigma_{max}}{b} \int y^2 dA$$

$\frac{\sigma_{max}}{b}$ → constant

$$\Rightarrow \boxed{\sigma_{xx} = \frac{M_z y}{I_z}} \qquad \text{where } \boxed{I_z = \int y^2 dA}$$

~~if y = ±~~



$$\boxed{\sigma_{yy} = \frac{M_x z}{I_x}}$$

new Continue

if $y = \pm b \Rightarrow \sigma = \sigma_{max} = \frac{Mb}{I}$

$$\Rightarrow \sigma_{max} = \frac{M}{(I/b)}$$

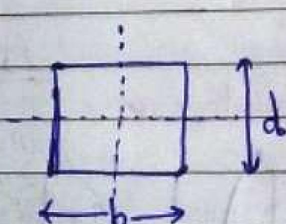
here $I/b \rightarrow$ (Geometric Property of body)

let $\frac{I}{b} = Z \Rightarrow \boxed{\sigma = \frac{M}{Z}}$

$Z \downarrow$ Section Modulus

[Note → we want if $Z \rightarrow max$ then $\sigma = min$]

Ex:



$I = \frac{1}{12} bd^3 \Rightarrow y_{max} = \frac{d}{2}$ (max)

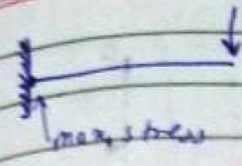
$$\frac{I}{y} = \frac{1}{12} bd^3 \times \frac{1}{d/2} \Rightarrow$$

$\Rightarrow \boxed{Z = \frac{1}{6} bd^2}$ if d is large the section Modulus will be larger (?)

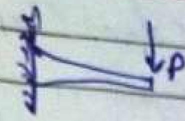
∵ $A = bd$ and Z should be ↑

if ~~b>>d~~ $d >> b$ the shape will be

↳ like rail line



then

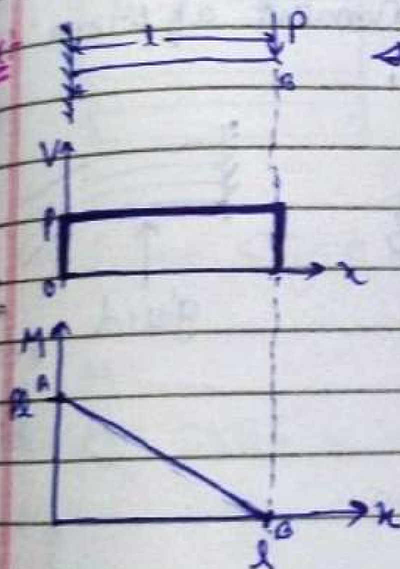


* If Z will be higher the stresses induce in the beam will be lower.

← Section Modulus →

→ section modulus should large for structural steel beams.
→ To reduce the mass and stress · induce in the body will be lower.

Ex:



← Cantilever

* Moment →

TYPE → ① mass moment of inertia — distribution of mass property of body to resist the rotation.

② Polar second moment of inertia

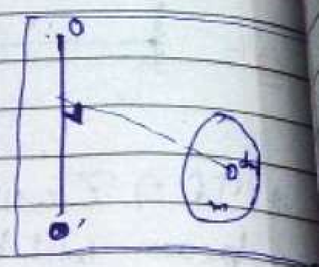
③ Moment of inertia of · an area

Q what exactly do they mean?
Q what is their physical interprtation?

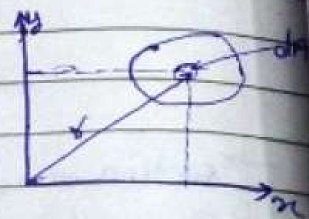
The again the points for Moment Type. →

① $\tau = I\alpha = M$ ← moment as $F = ma$

$\Rightarrow I = \int r^2\, dm = mk^2$

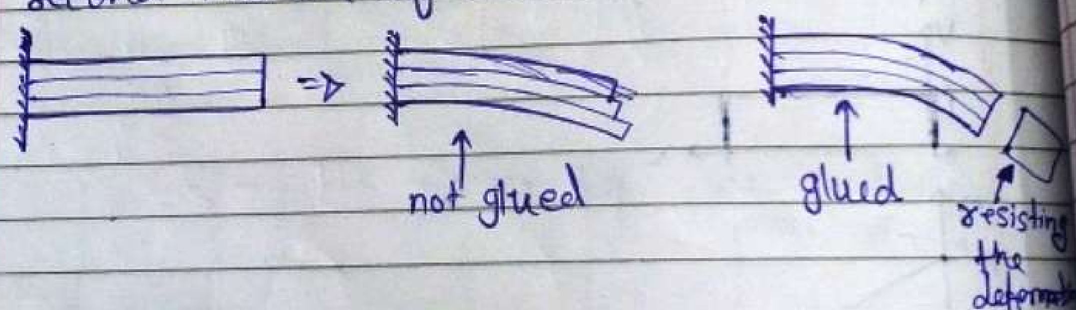


② Resistance of twisting $I_p = \int r^2\, dA$



③ $I_x = \int_A y^2\, dA$ or $I_y = \int_A x^2\, dA$

Known as the "second moment of Area" or second Moment of inertia",



Geometric property of the area

- Centroid →

$$x = \int_A x\, dA \Big/ \int_A dA$$

- Composite area →

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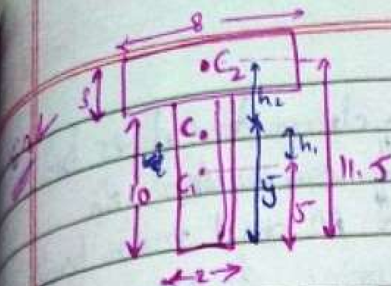


$$\bar{y} = \frac{\overset{A_1 \times c_1}{20 \times 5} + \overset{c_2 \times A_2}{11.5 \times 24}}{\underset{A_1 + A_2}{20+24}} = \frac{100+271}{44}$$

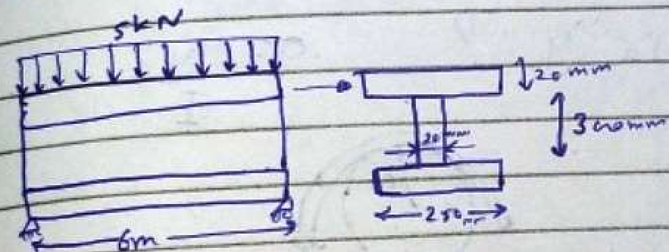
$$\Rightarrow \bar{y} = 8.54$$

$$I_{C_I} = I_{C_1} + h_1^2 A_1$$
$$I_{C_{II}} = I_{C_2} + h_2^2 A_2$$

[Parallel axis theorem]

* Design of beam for bending stresses →

Ex:- Absolute Max bending stress:

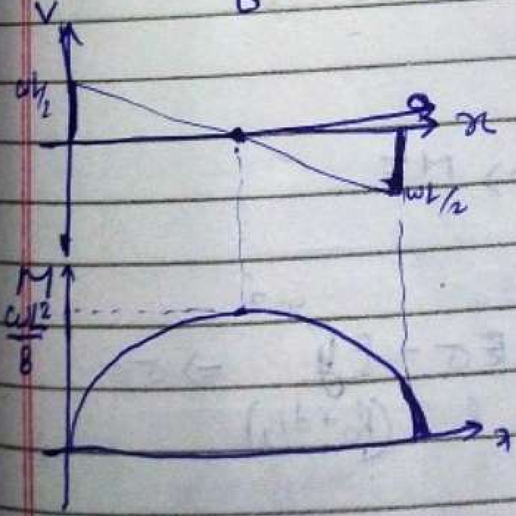


$\Sigma f_x = 0, \Sigma f_y = 0 \Rightarrow \boxed{R_A + R_B = 5} \Rightarrow M_A = 5 \times 3 - R_B \times 6 = 0$

$\Rightarrow R_B = 2.5$

$R_A = 2.5$

$$M_{max} = \frac{5 \times 36}{8} = \frac{Wl^2}{8} \Rightarrow M = 22.5$$



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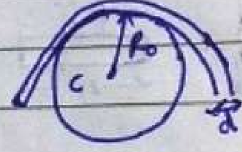
Q. A high - strength steel wire of diameter d is bent around a cylindrical drum of ~~rra~~ radius R_0. Determine the bending Moment M and max. bending stress σ_{max} in the wire assuming $d = 4mm$ and $R_0 = 0.5m$, $E = 200 GPa$ and proportional limit $\sigma_{p} = 1200 Mpa$

Soln strategy
$\Rightarrow$ What is given that can be used to get bending moment?

Soln step ① What is "y"? $\sigma = \frac{My}{I}$

$\Rightarrow y = 2 \because d = 4$

$I = \frac{\pi}{64} d^4$



$M = ?$

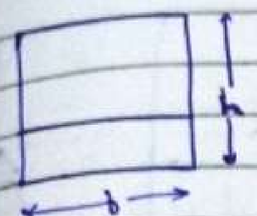
$\varepsilon = \frac{y}{\rho} \Rightarrow \sigma = E\varepsilon \Rightarrow \boxed{\sigma = \frac{Ey}{\rho}}$ — ① and $\boxed{\sigma = \frac{My}{I}}$ — ②

$$\Rightarrow \boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{\rho}}$$

$\Rightarrow M = \frac{EI}{\rho} \Rightarrow M = \frac{EI}{(R_0 + d/2)} \Rightarrow M =$

$\sigma = \underbrace{\frac{EI}{(R_0 + d/2)}}_{M} \times \frac{y}{I} \Rightarrow E\sigma = \frac{Ey}{(R_0 + d/2)}$ ($y = 2$) $\Rightarrow \sigma =$

Composite beam → Beams constructed of two or more diff. materials are refferred to as composite beams.



Where is Neutral axis?

∵ for nutral axis $\Sigma F = 0$

$\Rightarrow F_1 + F_2 = 0$

$$\boxed{\because \frac{\sigma}{y} = \frac{M}{I} = \frac{E}{\rho}} \quad (1) \Rightarrow \sigma = \frac{Ey}{\rho} \ \& \ F = \sigma A$$

$$\Rightarrow F_1 = \frac{E_1}{\rho}\int y_1 \, dA_1 \quad \text{and} \quad F_2 = \frac{E_2}{\rho}\int_2 y \, dA_2$$

$$\Rightarrow \frac{E_1}{\rho}\int_{A_1} y \, dA + \frac{E_2}{\rho}\int_{A_2} y \, dA = 0 \Rightarrow E_1\int_1 y \, dA + E_2\int_2 y \, dA = 0$$

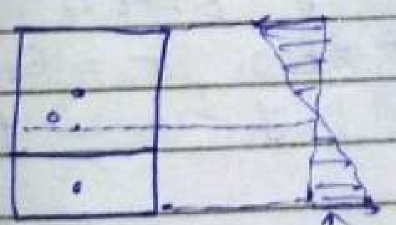
$$\Rightarrow \boxed{E_1 \bar{y}_1 A_1 + E_2 \bar{y}_2 A_2 = 0} \quad (2)$$

$\bar{y}_1$: N.A. of area (1) $\quad$ $\bar{y}_2$: N.A. of Area (2)

$\because \varepsilon = \dfrac{-y}{\rho}$ → Not depends on material property

Let N.A. of the whole beam is passing threw the point O then

~~suppose $E_1 > E_2$~~



for homogeneous

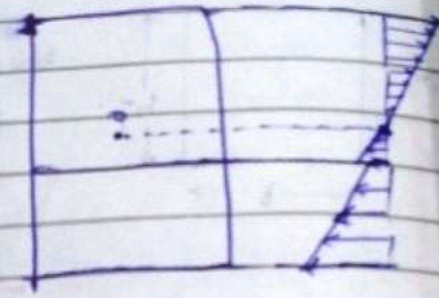
~~If $E_2 > E_1$~~

$\sigma_2 > \sigma_1$ for constant ε

$E_2 > E_1$

if $E_2 < E_1$

Suppose $E_2 > E_1$
$\sigma_2 > \sigma_1$
for Constant ε



Now

$$\because M = \int \sigma \, dA \times y$$

$$= \int_{A_1} \sigma dA y + \int_{A_2} \sigma dA y \qquad \because \sigma = \frac{E_1 y}{\rho}$$

$$= \int_{1} \frac{E_1 y}{\rho} y dA + \int \frac{E_2 y}{\rho} y dA$$

$$M = \frac{E_1}{\rho}\int_{1} y^2 dA + \frac{E_2}{\rho}\int_{2} y^2 dA \Rightarrow M = \frac{E_1 I_1}{\rho} + \frac{E_2 I_2}{\rho}$$

$$\Rightarrow \boxed{M = \frac{E_1 I_1 + E_2 I_2}{\rho}} \Rightarrow \frac{1}{\rho} = \frac{M}{E_1 I_1 + E_2 I_2}$$

$$\because \frac{\sigma}{y} = \frac{M}{I} = \frac{E}{\rho} \qquad \Rightarrow \sigma_1 = \frac{E_1 y_1}{\rho}, \; \sigma_2 = \frac{E_2 y_2}{\rho}$$

$$\Rightarrow \boxed{\sigma_1 = \left(\frac{-M}{E_1 I_1 + E_2 I_2}\right) E_1 y_1}, \; \boxed{\sigma_2 = \frac{-M}{E_1 I_1 + E_2 I_2} E_2 y_2}$$

~~making transforms~~ into
Homogeneous Material

let $\frac{E_2}{E_1} = n$ then in (*)

$$\bar{y}_1 A_1 + \frac{E_2}{E_1}\,\bar{y}_2 A_2 = 0 \Rightarrow \boxed{\bar{y}_1 A_1 + n\bar{y}_2 A_2 = 0} \quad -(3)$$

Note → If I ~~want to~~ have a Composite beam of two diff. materials. I can made it from one material only. it should satisfy the eqn —(3) and —(4)

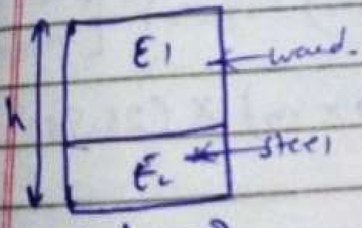
$$\text{if } M = 0 \Rightarrow E_1 I_1 + E_2 I_2 = 0 \Rightarrow \boxed{I_1 + n I_2 = 0} \quad -(4)$$

But during the transformation we keep the "h" constant. So, ~~one~~ the Beam is width will be ~~decr~~ reduced or increased.

Ex →

$E_w < E_s$

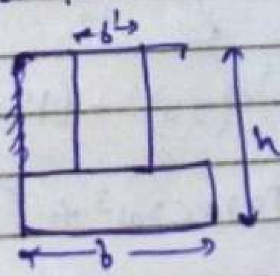
$E_1 < E_2$



$n = \frac{E_s}{E_w} = \frac{E_2}{E_1}$

(1)

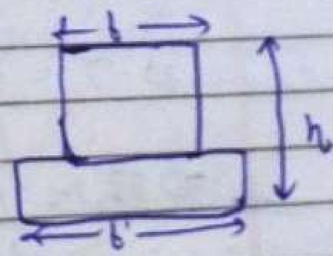
only steel



$\frac{b}{b'} = n$

$\sigma_w = \sigma_s / n$

(2)



$b'/b = n$

$\sigma_s = n\sigma_w$

(3)

Here, in all three cases strength is same

$\sigma_{steel} > \sigma_{wood} \Rightarrow \{\sigma = E\varepsilon$ for same ε

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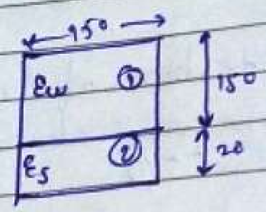
Q. Determine the normal stress at point B and C of Composite beam shown.

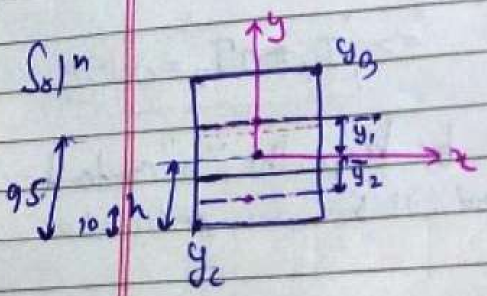


$E_\omega = 12$ GPa $\quad (E_s > E_\omega)$

$E_s = 200$ GPa

$M_x = -2$ kN.m

Sol^n



$\because E_1\bar{y}_1A_1 + E_2\bar{y}_2A_2 = 0$

$\Rightarrow 12(95-h)\times150\times150 + E_2(-h+10)\times \ldots$

$\Rightarrow \boxed{h = 36.38}$

$$\boxed{I = \frac{1}{12}bd^3}$$

$\Rightarrow I_1 = \frac{1}{12}(150)\times(150)^3 + (150\times150)(58.62)^2$ [Parallel axis theorem]

$\Rightarrow I_2 = \frac{1}{12}(150)\times(20)^3 + (150\times20)\times(26.38)^2$

$I_T = I_1 + I_2$

$\because \sigma_B = \frac{MY_1E_1}{E_1I_1+E_2I_2}$

$y_1 = y_B = 133.62$

$y_2 = y_C = 36.38$

$\sigma_B = 1.71$ MPa

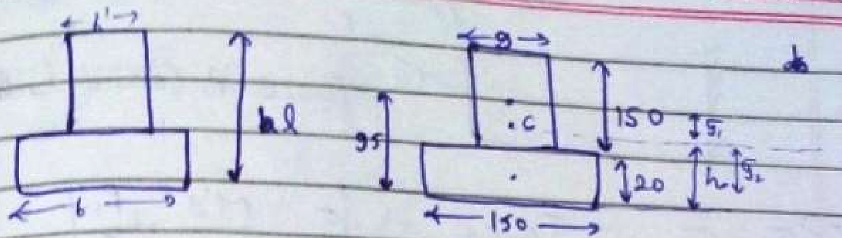
$\sigma_C = 7.78$ MPa

Q. Solve this using area transformation →

[Either convert whole beam in steel or wood.]

$n = \frac{100}{12}$ or $\frac{12}{100}$

only steel $\Rightarrow E_1 = 200\,GPa$



$b' = \frac{b}{n} = \frac{150 \times 12}{200} = 9$

$n = \frac{E_1}{E_2} = \frac{200}{12}$

$\Rightarrow (95 - h)(150 \times 9) + n(-h + 10)(150 \times 20) = 0$

$\Rightarrow \boxed{h = 12.235}$ mm

$\Rightarrow I_1 = \frac{1}{12}(9)(150)^3 + (150 \times 9)(82.765)^2 = 11.77\, mm^2$

$I_2 = \frac{1}{12}(150)(20)^3 + (150 \times 20)(2.235)^2 = 1.215\, mm^2$

$I_{total} = 12.985\, mm^2$

$\Rightarrow \sigma_s = \frac{\cancel{2}\, kN.m \times 157.765\,(mm) \times \cancel{200\,(GPa)}}{\cancel{2} \times \cancel{200\,(GPa)} \times 12.985\, mm^2}$

$\Rightarrow \sigma_s = \frac{157.765}{12.985}\, MPa \Rightarrow \sigma_s = 12.15\, MPa.$

$\sigma_w = \frac{\sigma_s}{n} = 0.729\, MPa$

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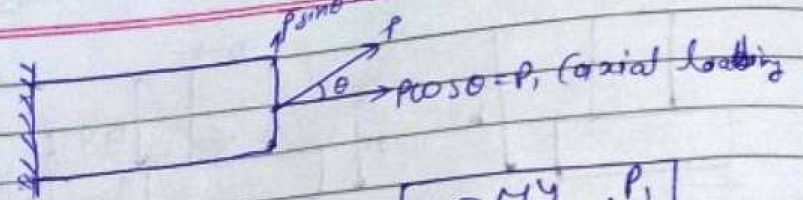
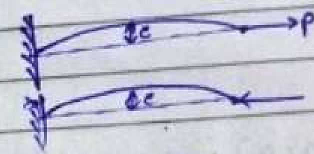
Q4



$$\Rightarrow \sigma = \sigma_{bend} + \sigma_{Axis} \Rightarrow \boxed{\sigma = \frac{My}{I} + \frac{P_1}{A}}$$

Q Buggling



$M = Pe$. (it will go out of the plane)

Note

assuming the thickness of the beam is "b".

in ③ there should be induced τ to supporting or balancing

$\sigma = \frac{My}{I}$

$dA\,\sigma = \frac{dM\,y\,dA}{I}$

$\cancel{df} = \frac{dMy\,dA}{I}$

$$\Rightarrow F + F_2 = F + dF \Rightarrow F_2 = dF$$

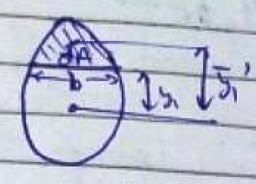
$$\Rightarrow (b\,dx)\,\tau = \frac{dMy}{I}\,dA$$

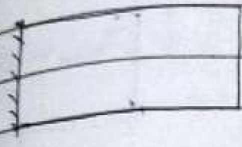
$$\Rightarrow \tau = \frac{1}{Ib}\frac{dM}{dx}\int y\,dA$$

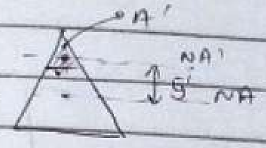
$$\Rightarrow \tau = \frac{V}{Ib}\int y\,dA \qquad \text{let } \int y\,dA = Q$$

Note: y is the distance ~~btw~~ the whole cross section's N.A. ~~and to~~ and N.A. of the element dA.

$$\Rightarrow \boxed{\tau = \frac{VQ}{Ib}}$$ ☆

If cross section is 





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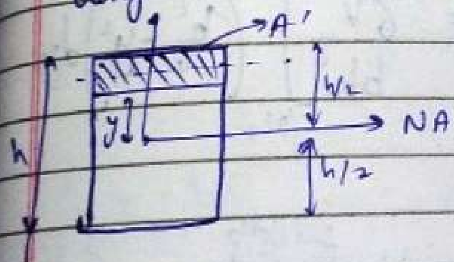
$$\tau = \frac{VQ}{It} \quad t \rightarrow \text{thickness}$$

$$\Rightarrow \tau \times t = \frac{VQ}{I}$$

$$\Rightarrow \boxed{q = \frac{VQ}{I}} \text{ (shear flow)}$$

→ due to presence of transverse shear stresses the longitudinal shear will also be present.



$I = \frac{1}{12} b h^3$, $t = b$ (thickness)

$\therefore Q = \bar{y}' A'$ & $A' = b \times \left(\frac{h}{2} - y\right)$

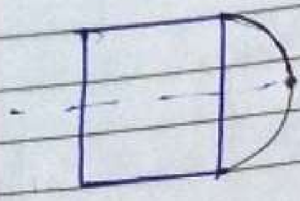
$$\bar{y}' = y + \frac{1}{2}\left(\frac{h}{2} - y\right)$$

$$\Rightarrow \tau = \frac{V}{b}\left[\frac{\overset{A'}{b\left(\frac{h}{2} - y\right)} \times \overset{\bar{y}'}{\left[y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right]}}{\frac{1}{12} b h^3}\right]$$

$$= \frac{12V}{b^2 h^3}\left[b\left(\frac{h}{2} - y\right) \cdot \frac{1}{2}\left(\frac{h}{2} + y\right)\right]$$

$$= \frac{12V}{b^2 h^3} \times \frac{b}{2}\left[\frac{h^2}{4} - y^2\right] = \frac{6V}{bh^3}\underbrace{\left[\frac{h^2}{4} - y^2\right]}_{\text{eqn is parabolic}}$$

So, distribution will be → at $y = h/2$, $\tau = 0$



τ_{max} (at $y=0$) at $y = 0$, $\tau = \tau_{max} = \frac{6V}{bh^3}$

$$\boxed{\tau_{max} = \frac{3}{2}\left(\frac{V}{bh}\right)}$$

Note: $V \rightarrow$ shear

$\frac{V}{ab} = \frac{V}{\text{area}}$ = average shear stress

$$\boxed{\tau_{max} = \frac{3}{2}\tau_{avg}}$$

∵ τ is induced by V so integration of τ should give me "V".

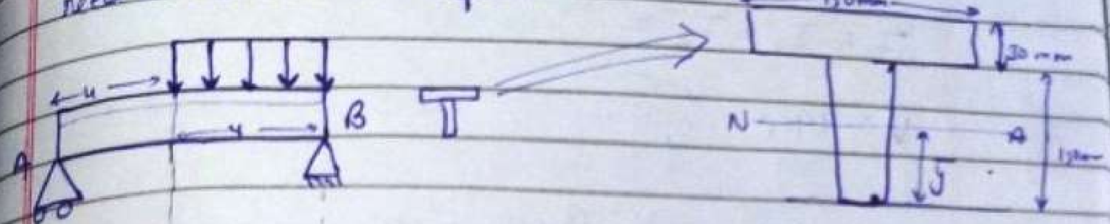
let's $$k = \int_{-h/2}^{h/2} \tau \, dy = \frac{6V}{bh^3}\int_{-h/2}^{h/2}\left(\frac{h^2}{4} - y^2\right) dy$$

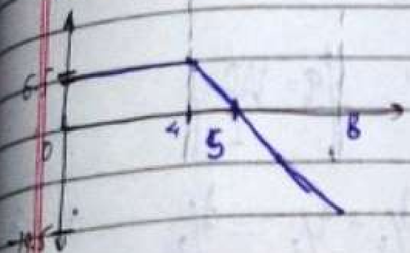
$\Rightarrow \boxed{k = \frac{V}{b}} \rightarrow$ V per unit ~~length~~ width.

∵ Maximum shear stress appears at the middle so crack appears at the middle.

Ex: The beam shown is made from two boards. Determine the max. shear stress in the glue necessary to hold the boards together along the ~~se~~ beam where they are joined.

To be able to apply shear formula, we need to know the distribution of shear force V across the beam. So first we need to draw SFD. ~~first~~.





$R_A = 6.5\,kN$, $R_B = 19.5\,kN$

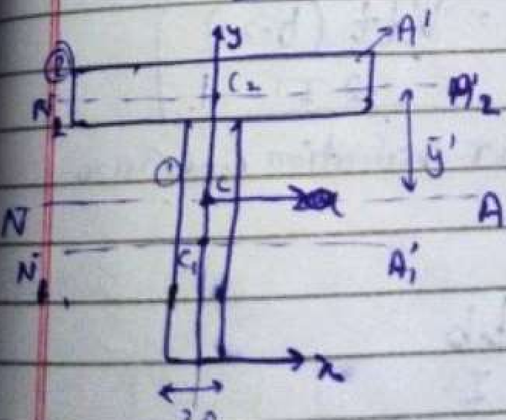
eqn $V = 6.5 - 6.5x$

at $V = 0$, $6.5 \times x = 6.5$

$x = 1$

$\Rightarrow \left|V_{max}\right| = 19.5$

$\because \tau = \dfrac{VQ}{It}$, $Q = ?$, $I = ?$, $t = ?$



$t = 30mm$, $Q = \bar{y}'A'$ (above NA)

$= 45 \times 150 \times 30$

~~$\bar{y} = y_1 A_1 +$~~ $= 102.025\,mm$

$\because \bar{y} = \dfrac{y_1 A_1 + y_2 A_2}{A_1 + A_2}$

$= \dfrac{75 \times 30 \times 150 + 165 \times 30 \times 150}{30 \times 150 + 30 \times 150}$

$= \dfrac{75 + 165}{2} \Rightarrow \boxed{\bar{y} = 120}$

w.r.t. to NA (Parallel axis theorem)

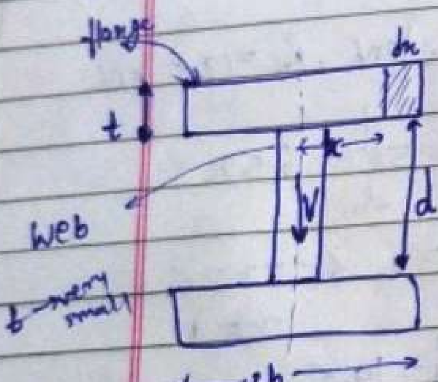
$I_1 = \frac{1}{12} \times (30)(150)^3 + (150 \times 30) \times (45)^2$

$I_2 = \frac{1}{12} \times (150) \times (30)^3 + (150 \times 30) \times (45)^2$

$I = I_1 + I_2 =$

$V = 19.5$ Sycl

~~#~~ Shear flow



$Q = \bar{y}' A'$, $\boxed{q = \frac{VQ}{I}}$

$t << b$

$\bar{y}' = \frac{d}{2} + \frac{t}{2} \simeq \frac{d}{2}$

$A' = (b-x)t$

$Q = \frac{d}{2}[(b-x)t]$

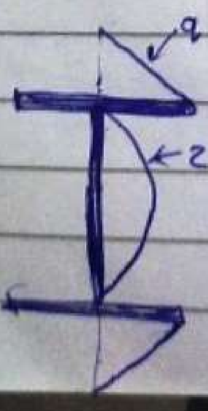
$q = \frac{V \frac{d}{2} t (b-x)}{I}$ $\Rightarrow$ $\boxed{q = \frac{Vdt(b-x)}{2I}}$

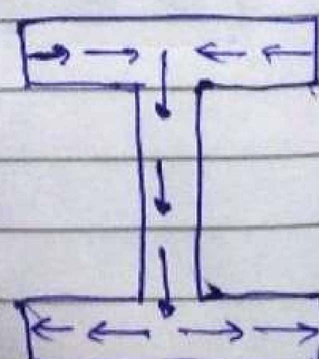
linear variation w.r.t. to x

at $x = b$, $q = 0$

at $x = 0$, $q = q_{max} = \frac{Vdtb}{2I}$

total variation





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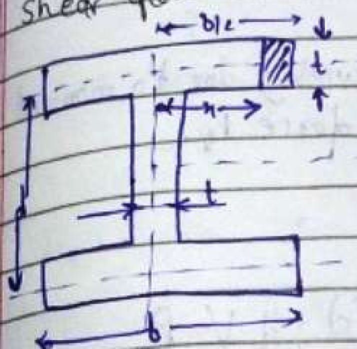
shear flow in thin walled Member →



$$q = \frac{VQ}{I}, \quad Q = \bar{y}' A', \quad \bar{A} = \left(\frac{b}{2} - x\right) t$$

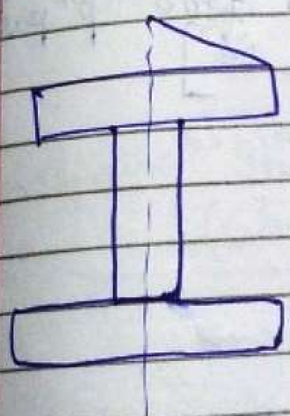
$$\bar{y}' = d/2$$

$$q = \frac{V \times d/2 \, (b/2 - x)\, t}{I}$$

$$\Rightarrow \boxed{q = \frac{Vdt\,(b/2 - x)}{2I}}$$

at $x = b/2$, $q = 0$

at $x = 0$, $q = \frac{Vdtb}{4I}$



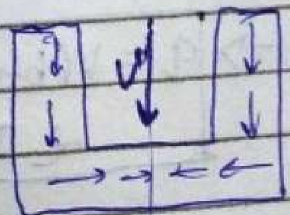
force in the flange

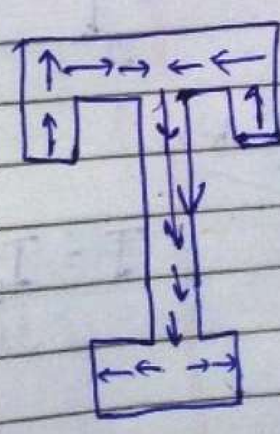
$$F_f = \int_0^{b/2} q\,dx = \frac{Vdt}{2I} \int_0^{b/2} \left(\frac{b}{2} - x\right) dx$$

$$\boxed{F_f = \frac{Vdt\,b^2}{16I}}$$

force in the web $\Rightarrow F_w = V$

* shear flow in some common cases →



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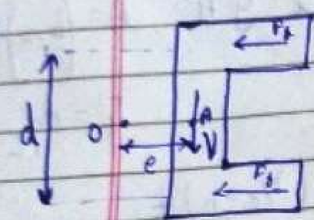
Shear Center → ⑦ [theory]

there will be twisting due to moment generated by force F_f.



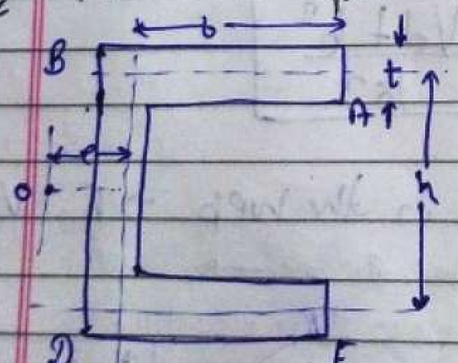
$$\Rightarrow \boxed{V \times e = F_f \times d}$$

$$\Rightarrow e = \frac{F_f d}{V} = \frac{Fd}{P}; \text{ if } V = P$$

O → shear center V [only bending not twisting if we apply force "p" passing through it]

* Shear center for Open thin walled member →

Q Find shear flow and shear center →



$t \ll b < h$

$Q = \bar{y}'A'$, $A' = (b-x)t$

$= \frac{h}{2}(b-x)t$ $\bar{y}' = h/2$

$q_v = \frac{VQ}{I}$

$$\Rightarrow \boxed{q = \frac{V(b-x)t \times h}{2I}}$$

$I = I_w + 2I_f$ =

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$I_w = \frac{1}{12} t h^3$, $I_f = \frac{1}{12} b t^3 + (b \times t) \cdot \left(\frac{h}{2}\right)^2$

$I = I_w + 2 I_f \Rightarrow \boxed{I = \frac{1}{12} t h^2 (6b + h)}$

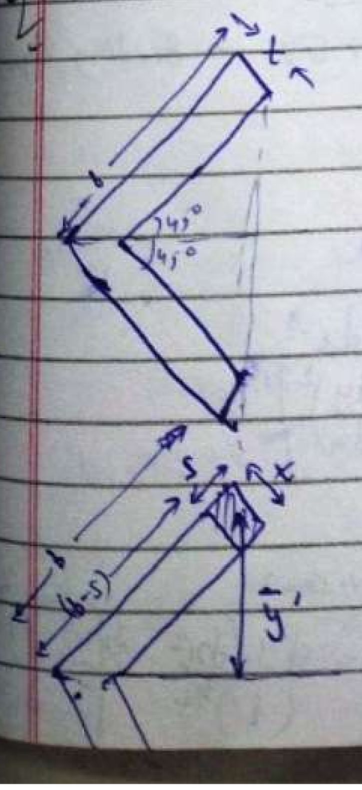
$\therefore F_f = \int q \, dx = \int_0^b \frac{V t h}{2I} (b - x) \, dx$

$\Rightarrow \boxed{F_f = \frac{V t h b^2}{4I}}$

$\because e = \frac{F_f d}{V}$ $\because d = h \Rightarrow e = \frac{t h^2 b^2}{4I}$

$\Rightarrow \boxed{e = \frac{3b^2}{(6b + h)}}$ $\Rightarrow$ e depends on the geometry of the cross section, not depends on the loading

Q Determine the location of shear center and F_f.



[$\because$ force lines will cross at the peak of the structure or beam $\Rightarrow$ shear center should lie on that.

$Q = \bar{y}' A'$
$= \left(b - \frac{s}{2}\right) \sin 45^\circ \times s t \Rightarrow Q = \left(b - \frac{s}{2}\right) \frac{s t}{\sqrt{2}}$

$F_f = q = \frac{QV}{I} = \frac{V (b - s/2) s t}{\sqrt{2} I}$

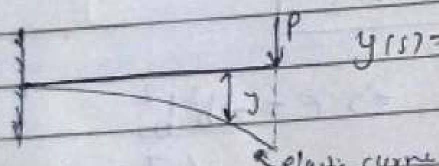
$$\because F_f = \int q\, ds = \frac{Vt}{\sqrt{2} I} \int_0^b \left[bs - \frac{s^2}{2} \right] ds$$

$$\because I = \frac{1}{3} t b^3 \quad (2)$$

$$\Rightarrow F_f = \frac{1}{3\sqrt{2}} \frac{V t b^2}{I}$$

$$\Rightarrow \boxed{F_f = \frac{V}{\sqrt{2}}}$$

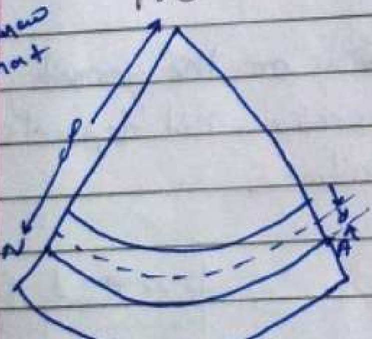
* Deflection of Beams



$y(s) = ?$ How can I relate "y" with M, V and W ← distributed load density.

fig (1)

$\because$ we know that



$$\varepsilon = -y/\rho \quad \text{and} \quad \boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{\rho}}$$

$$\Rightarrow \boxed{\frac{M}{EI} = \frac{1}{\rho}} \quad (1)$$

here $EI \rightarrow$ Bending Rigidity

$EA \rightarrow$ Axial rigidity

Rigidity $\rightarrow$ property of material.

Now

$\because$ We know that

Radius of curvature $\Rightarrow \dfrac{1}{\rho} = \dfrac{d^2y/dx^2}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$

for fig (1) if deformation is very small

then $\left(\dfrac{dy}{dx}\right)$ will be v. v. small.

$$\Rightarrow \frac{1}{\rho} = \frac{d^2y/dx^2}{(1)^{3/2}} \Rightarrow \boxed{\frac{1}{\rho} = \frac{d^2y}{dx^2}} \quad (2)$$

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force density

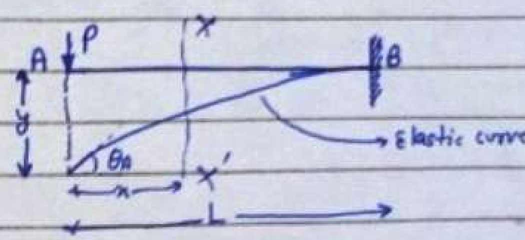
from -① and -② $\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}}$ $\because V = \frac{dM}{dx}$ & $\omega = \frac{dV}{dx}$

$\Downarrow$

$\Rightarrow \boxed{\frac{d}{dx}\left[EI\frac{d^2y}{dx^2}\right] = V} \Leftarrow \boxed{EI\frac{d^2y}{dx^2} = M} \Rightarrow \boxed{\frac{d^2}{dx^2}\left[EI\frac{d^2y}{dx^2}\right] = W}$

Note — this ~~at~~ all for only isotropic, homogeneous and it beams are prizmatic.

General Eq:-



$$EI\frac{d^2y}{dx^2} = -Px \quad -①$$

$$EI\frac{dy}{dx} = \frac{-Px^2}{2} + C_1 \quad -② \quad \text{and} \quad EIy = \frac{-Px^3}{6} + C_1x + C_2 \quad -③$$

Boundry Condition → at fixed end $\because$ it is not going to deform $y = 0$ and $dy/dx = 0$

$\Rightarrow$ at $x = L$, $y = 0$ and at $x = L$, $dy/dx = 0$

$$\Rightarrow 0 = \frac{-PL^3}{6} + C_1L + C_2 \qquad 0 = \frac{-PL^2}{2} + C_1$$

$$\Rightarrow C_2 = \frac{PL^3}{6} - \frac{PL^3}{2} \Rightarrow \boxed{C_2 = \frac{-PL^3}{3}} \qquad \Rightarrow \boxed{C_1 = PL^2/2}$$

the ~~(EI)y~~ $(EI)y = \frac{-Px^3}{6} + \frac{PL^2}{2}x - \frac{PL^3}{3}$

for $y = y^{max}$ at $x = 0$

$$\Rightarrow y^{max}(EI) = \frac{-PL^3}{3} \Rightarrow \boxed{|y^{max}| = \frac{PL^3}{3EI}}$$ downwards

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NOTEBOOK

~~at x~~

$$\boxed{\frac{dy}{dx} = \frac{P}{2EI}(L^2 - x^2)}$$

at $x = 0$

$$\boxed{\left.\frac{dy}{dx}\right|_{max} = \frac{PL^2}{2EI}}$$

07/11/23

ME 2Ə

Ex:-

→ will be taken as a single term [Use $(x-L/2)=x'$ then $\int x'dx = \frac{(x')^2}{2} = \frac{(x-L/2)^2}{2}$]

$$\Rightarrow EI\frac{d^2y}{dx^2} = R_A x \;\vdots\; -P(x-L/2) \quad (1) \;\because \text{this is applied after } x = L/2$$

$R_A = P/2 = R_B$ by $\sum F_y = 0$ & $M_A = 0$

$$\Rightarrow EI\frac{dy}{dx} = R_A\frac{x^2}{2} \;\vdots\; -P\frac{(x-L/2)^2}{2} + c_1 \quad (2)$$

$$\Rightarrow EIy = R_A\frac{x^3}{6} \;\vdots\; -P\frac{(x-L/2)^3}{6} + c_1x + c_2 = 0$$

$\because$ at $x=0$, $y=0$,

$\Rightarrow \boxed{0 = c_2}$

at $x = L$, $y = 0$

$$\Rightarrow 0 = \left(\frac{P}{2}\right)\left(\frac{L^3}{6}\right) - \frac{P(L/2)^3}{6} + c_1L + 0$$

$$\Rightarrow 0 = \frac{PL^3}{12} - \frac{PL^3}{48} + c_1L$$

$$\Rightarrow \boxed{c_1 = -\frac{PL^2}{16}}$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{P}{2}\frac{x^2}{2} - \frac{P(x-L/2)^2}{2} + \left(\frac{-PL^2}{16}\right)$$

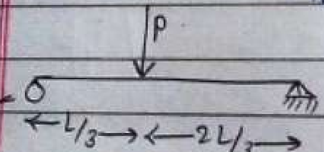
for $\frac{dy}{dx} = 0$, $\Rightarrow 0 = \frac{Px^2}{4} - \frac{P}{2}\left[x^2 + \frac{L^2}{4} - xL\right] - \frac{PL^2}{16}$

$$\Rightarrow -\frac{x^2}{4} - \frac{3L^2}{16} + \frac{L}{2}x = 0$$

$$\Rightarrow 4x^2 + 3L^2 - 8Lx = 0 \Rightarrow x = \frac{L}{2}, \frac{3L}{2}$$

out of range or boundary.

Ex:-



$P = R_A + R_B$ — (1)

$M_A = 0 = -\frac{PL}{3} + R_BL$

$\Rightarrow R_B = P/3$

$R_A = 2P/3$

$$\because EI\frac{d^2y}{dx^2} = R_Ax \;\vdots\; -P\left(x - L/3\right)$$

$$\Rightarrow EI\frac{dy}{dx} = R_A\frac{x^2}{2} \;\vdots\; -\frac{P(x-L/3)^2}{2} + c_1$$

$$\Rightarrow EIy = R_A\frac{x^3}{6} \;\vdots\; -\frac{P(x-L/3)^3}{6} + c_1x + c_2$$

Boundary condition →

at $x=0 \Rightarrow y=0$

$\Rightarrow \boxed{0 = C_2}$

at $x = L,\ y = 0 \Rightarrow 0 = \frac{PL^3}{9} - \frac{P}{6}\left(\frac{2L}{3}\right)^3 + C_1 L + \cancel{C_2}$

$\Rightarrow \frac{PL^3}{9} - \frac{8PL^3}{162} + C_1 L = 0$

$\Rightarrow -\frac{9PL^2}{81} + \frac{4PL^2}{81} = C_1 \Rightarrow \boxed{C_1 = \frac{-5PL^2}{81}}$

for $dy/dx = 0$

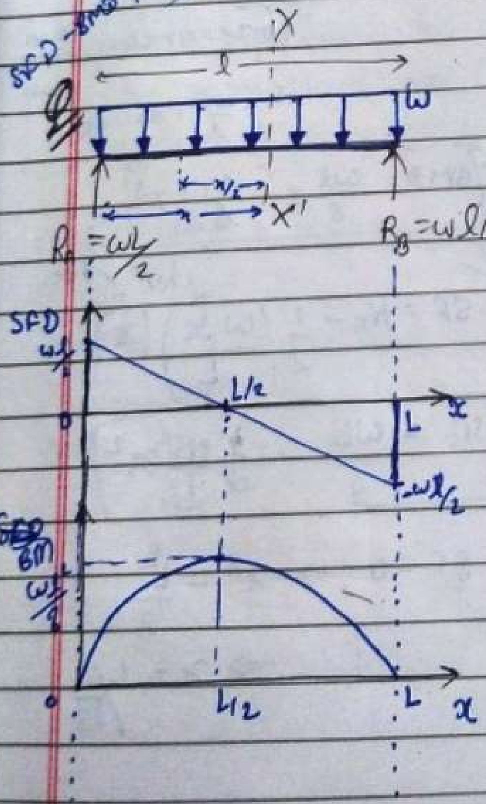
$\Rightarrow 0 = \frac{Px^2}{3} - \frac{P}{2}(x - L/3)^2 - \frac{5PL^2}{81}$

$\Rightarrow \boxed{27x^2 - 54Lx + 19L^2 = 0} \Rightarrow x = L(1 \pm 0.544)$

$\Rightarrow \boxed{x = 0.456\,L}$ Ans

[other value is > L]

SFD-BMD problem



$(SF) = \frac{wl}{2} - wx = 0$, at $x = l/2$, $SF = 0$

$(BM) = \frac{wl}{2}x - \frac{wx^2}{2}$ $\left(\frac{1}{2}wx\right)\left(\frac{x}{2}\right)$

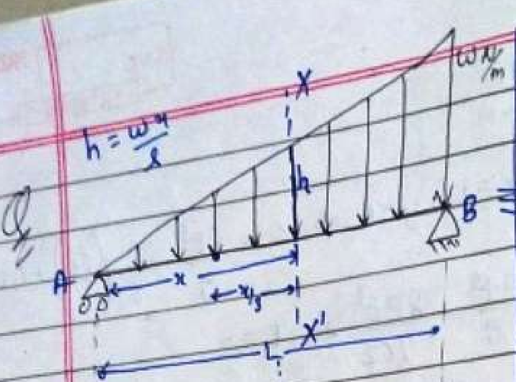
R_A distance of R_A from cross section; force; distance of concentrated load from cross section

$M = (BM) = \frac{wl}{2}x - \frac{wx^2}{2}$ (Parabola)

$\frac{dM}{dx} = \frac{wl}{2} - wx = 0 \Rightarrow x = l/2$

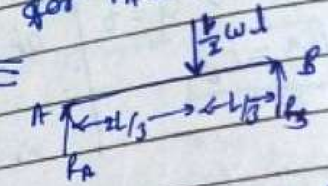
at $x = l/2$, $M = BM = \frac{wl^2}{4} - \frac{wl^2}{8}$

$\Rightarrow \boxed{M_{max} = BM = \frac{wl^2}{8}}$

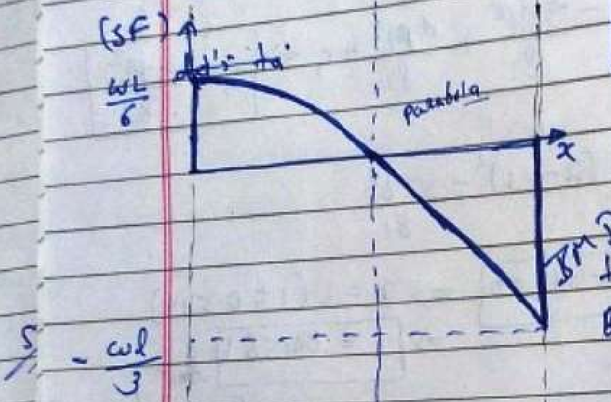


~~$R_A = \frac{wl}{6}$ and $R_B = \frac{wl}{3}$~~

for R_A & R_B



$\Rightarrow R_A = \frac{wl}{6}$ and $R_B = \frac{wl}{3}$



Now

let's take a cross section cut at distance x from A.

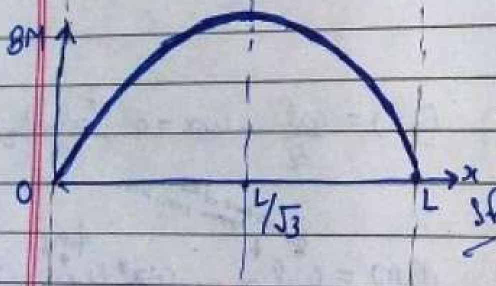
then (R_A) (distance from XX')

BMD

$$BMD = \frac{wl}{6}x - \underbrace{\frac{1}{2}\frac{x\,wx}{l}}_{\text{force = area} = \frac{1}{2}\times \text{distance (Base)} \times \text{hight}} \times \frac{x}{3}$$

$h = \frac{wx}{l}$, Base $= x$

$$\Rightarrow \boxed{BM = \frac{wl}{6}x - \frac{1}{6}\frac{w}{l}x^3}$$



SFD

$$SF = R_A - \frac{1}{2}\underbrace{\left(\frac{wx}{L}\right)}_{\text{hight}}\underbrace{(x)}_{\text{Base.}}$$

$$\boxed{SF = \frac{wL}{6} - \frac{1}{2}\frac{w}{L}x^2}$$

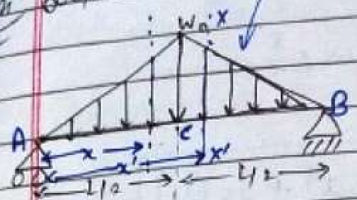
$SF = 0$, $\Rightarrow x^2 = \frac{L^2}{3}$

$\Rightarrow x = L/\sqrt{3}$

symmetric triangular loading

Bending

Ex: Determine the max. deflection for given simply supported beam.



total load $= 2\left[\frac{1}{2} \times w_0 \times \frac{L}{2}\right] \Rightarrow P = \frac{1}{2} w_0 L$

$\boxed{R_A + R_B = P} \Leftrightarrow \Sigma F_y = 0$

$M_A = 0 \Rightarrow 0 = \frac{1}{2} w_0 L \times \frac{L}{2} - R_B L$

$\Rightarrow \boxed{R_B = \frac{w_0 L}{4} = R_B}$ or $\boxed{R_A = R_B = \frac{P}{2}}$

$EI \frac{d^2y}{dx^2} = \cdot$

The condition of zero deflection at A and of zero slope at C do not require the general moment equation. Only the moment eqn for segment AC is needed.

$M_A = \left[R_A x - \overbrace{\frac{1}{2}(x)\left(\frac{wx}{L/2}\right)}^{force} \times \frac{x}{3}\right]$ force

$+ \left[\frac{1}{2} w_0 L - \frac{1}{2}(L-x)\left(\frac{w(L-x)}{L/2}\right)\right]$

$\frac{w x^2}{L}$

x distance from XX

$$EI \frac{d^2y}{dx^2} = M_{AC} = \frac{w_0 L}{4} x - \frac{1}{2}(x)\left(\frac{wx}{L/2}\right) \times \frac{x}{3}$$

$$\Rightarrow EI \frac{d^2y}{dx^2} = \frac{w_0 L}{4} x - \frac{w_0 x^3}{3L} \quad — (1)$$

$$\Rightarrow EI \frac{dy}{dx} = \frac{w_0 L x^2}{8} - \frac{w_0 x^4}{12L} + C_1 \quad — (2)$$

$$\Rightarrow EI (y) = \frac{w_0 L}{24} x^3 - \frac{w_0}{60L} x^5 + C_1 x + C_2 \quad — (3)$$

using boundary condition at $x = 0, y = 0 \Rightarrow \boxed{C_2 = 0}$

at $x = L/2, \frac{dy}{dx} = 0$ [because of symmetry]

in eqn (2)

$$\Rightarrow 0 = \frac{w_0 L^3}{32} - \frac{w_0 L^3}{192} + C_1 + 0 \Rightarrow \boxed{C_1 = \frac{-5 w_0 L^3}{192}}$$

$$\Rightarrow \cancel{EI\frac{dy}{dx} = w_0 L}$$

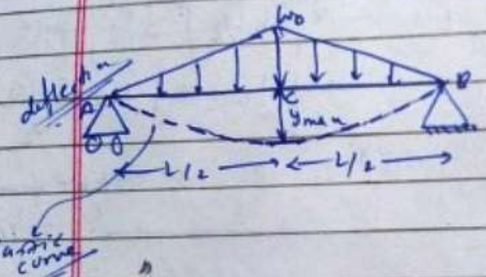
Now

$$\Rightarrow EI\,y = \frac{w_0 L}{24}x^3 - \frac{w_0}{60L}x^5 + \frac{-5}{192}w_0 L^3 x + 0$$

$$\Rightarrow \boxed{EIy = \frac{-w_0 x}{960L}\left[-40L^2x^2 + 16x^4 + 25L^4\right]}$$

$\because y_{max}$ is at $x = L/2$

$$\Rightarrow \boxed{y_{max} = \frac{1}{EI}\left(\frac{-w_0 L^4}{120}\right)}$$



Note 4 Condition → 4 known

→ $W = 0$ at $x = 0$ & $x = L$.

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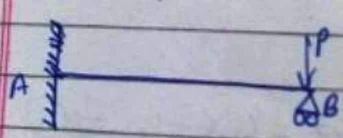
Statically Indeterminate Beams and shaft

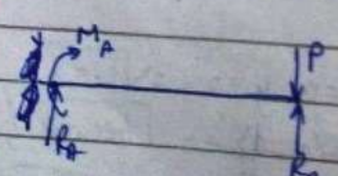
→ if no. of unknown rxn exceeds the available no. of eq of



$R_A + R_B = P$ —(1)

$\Sigma M_B = R_A \times L + M_A = 0$

FBD



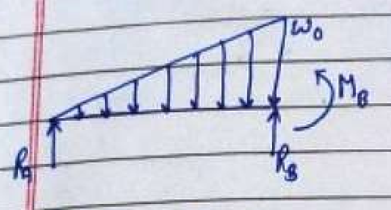
[∵ Why we are supported the beam or semthing

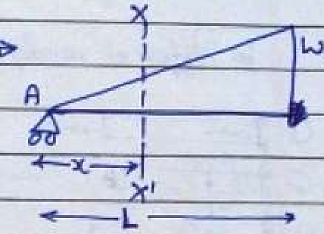
→ to increase stiffness]

Note

These unknowns can always be found from the boundary and/or continuity for the Problem.

Ex: Determine the Reaction at A. →





$\Sigma F_y = 0 \Rightarrow R_A + R_B = \frac{w_0}{2} \times L$ —① $\Sigma M_B = 0 \Rightarrow R_A \times L - \frac{w_0 L}{2} \times \frac{L}{3} = M_B$ —②

→ ~~the~~ 2 eqn & 3 ~~kn~~ unknown → how to solve ?

∵ We know that $EI\frac{d^2y}{dx^2} = M_x \Rightarrow EI\frac{d^2y}{dx^2} = R_A x - \left(\frac{1}{2}\left(\frac{w_0 x}{L}\right) x\right)\left(\frac{x}{3}\right)$

hight   Base   Force   distance from Grav section of

$\Rightarrow EI\frac{d^2y}{dx^2} = R_A x - \frac{w_0}{6L} x^3$ —③

$\Rightarrow EI\frac{dy}{dx} = R_A\frac{x^2}{2} - \frac{w_0}{24}\frac{x^4}{L} + C_1$ —④

$\Rightarrow EI\,y = R_A\frac{x^3}{6} - \frac{w_0 x^5}{120 L} + C_1 x + C_2$ —⑤

at $x=0$, $y=0$ $\Rightarrow \boxed{C_2 = 0}$, at $x = L$, $y = 0 \Rightarrow 0 = \frac{R_A L^3}{6} - \frac{w_0 L^4}{120} + C_1 L$ ⓐ

at $x = L$, $dy/dx = 0$

$\Rightarrow 0 = R_A\frac{L^2}{2} - \frac{w_0 L^3}{24} + C_1$ —ⓑ

$\boxed{C_1 = \frac{-w_0 L^3}{120}}$

$\Rightarrow$ ⓑ $\times L$ − ⓐ $\Rightarrow R_A L^3\left(\frac{1}{2} - \frac{1}{6}\right) - \frac{w_0 L^4}{24}\left(1 - \frac{1}{5}\right) = 0 \Rightarrow \boxed{R_A = \frac{w_0 L}{10}}$

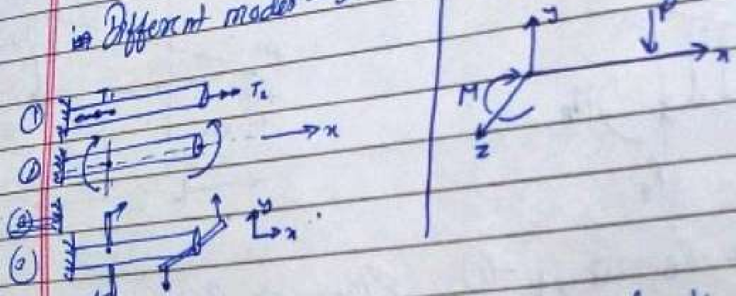
eq eq.

Chapter-4
Torsion [transforming the torque]

Ex- using a screw driver.

in Different modes →

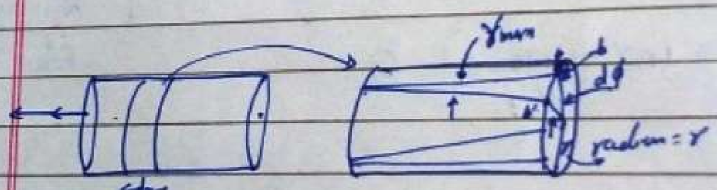
In Bending



We are remaining at a limit so that length of the object or length of the line does not change. only there is change in angle [OR the angle b/w the lines changes]

⇒ $\varepsilon_N = 0$ [Normal strain] and $\varepsilon_s \neq 0$ [shear strain]

Now

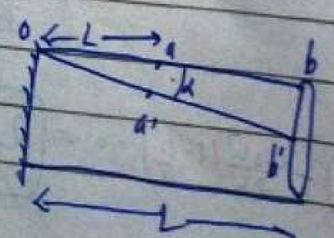


$\gamma_{max} = \frac{bb'}{ob} = \frac{r\,d\phi}{dx} = r\theta$ where $\theta = \frac{d\phi}{dx}$ = twist density or twist per unit length of the shaft

⇒ $\boxed{\gamma_{max} = r\theta}$

in general

$\boxed{\gamma = \rho\theta}$ ⇒ $\gamma = \frac{\rho}{r}\gamma_{max}$



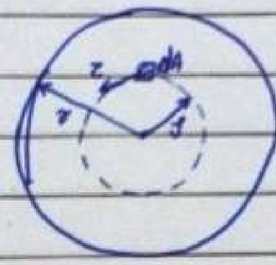
$\because$ stress $\tau = G\gamma' = G\rho\theta$

$\tau_{max} = G\gamma_{max} = Gr\theta$ $\longrightarrow$ $\boxed{\tau = \tau_{max}\dfrac{\rho}{r}}$

Note: at $r=0$, $\gamma_{max}=0$, $\tau_{max}=0$

Bars of linear elastic Material

$\because \boxed{\tau = \dfrac{\rho}{r}\,\tau_{max}}$



$dF = \tau\, dA$

$dM = \rho\, dF = \rho\,\tau\, dA$

$\Rightarrow dM = \rho \times \dfrac{\tau_{max}}{r} \cdot \rho\, dA \Rightarrow \boxed{dM = \dfrac{\tau_{max}}{r}\rho^2 dA}$

$\Rightarrow T = \int dM = \dfrac{\tau_{max}}{r}\int \rho^2 dA$, $\int \rho^2 dA = I_p = J \rightarrow$ Polar Moment of Area.

$\Rightarrow T = \dfrac{\tau_{max}}{r} I_p$ or $\boxed{\tau_{max} = \dfrac{Tr}{I_p}} \equiv \sigma = \dfrac{My}{I}$

$T \downarrow$ torque

[Torsion formula]